MA 16200
EXAM 3 Form 01
April 10, 2018

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

1. You must use a #2 pencil on the scantron.

2. Write 01 in the TEST/QUIZ NUMBER box on the scantron and bubble in the appropriate space below.

3. On the scantron, fill in your TA's name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

5. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.

6. Sign the scantron.

7. Fill in your name and your instructor's name on the question sheets above.

8. There are 12 questions, each worth 8 points (you will automatically earn 4 points for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–12. Do all your work on the question sheets.

9. Turn in both the scantron and the exam when you are finished.

10. If you finish the exam before 7:20, you may leave the room after turning in the scantron and the exam. If you don't finish before 7:20, you MUST REMAIN SEATED until your TA comes and collects your scantron and your exam.

11. Show your work and circle your answers on the exam. Although no partial credit will be given, any disputes about grades or grading can be settled by examining your written work on the question sheets.

12. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

1. For which values of c is $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2n}{n^2+c^2}\right)^n$ convergent?

A. All values of c.

B. $|c|<1$.

C. $|c|<2$.

D. $|c|>2$.

E. No values of c.

2. What are the radius of convergence R and interval of convergence I of the power series

$$\sum_{n=1}^{\infty}\frac{(x-2)^n}{\sqrt{n}\,3^n}?$$

A. $R=3$ and $I=(-1,5]$

B. $R=3$ and $I=[-1,5)$

C. $R=3$ and $I=(-1,5)$

D. $R=\infty$ and $I=(-\infty,\infty)$

E. $R=1$ and $I=(1,3)$

3. Which of the following is true about the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$?

A. Diverges by the comparison test using $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$.

B. Converges by the limit comparison test using $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$.

C. Diverges by the limit comparison test using $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$.

D. Converges by the Integral Test.

E. Diverges by the Integral Test.

4. For which of the following series is the ratio test conclusive (that is, it really determines whether or not the series converges)?

(i.) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

(ii.) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n^2}$

(iii.) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n(1+n)}$

(iv.) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3^n}$

A. (i) and (ii) only.

B. (iii) and (iv) only.

C. All of these series.

D. None of these series.

E. Another combination not listed above.

5. Find all of the series below that are convergent but **not** absolutely convergent.

(i.) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

(ii.) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(iii.) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

(iv.) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+1}$

A. (i) and (ii) only.

B. (iii) and (iv) only.

C. All of these series.

D. None of these series.

E. Another combination not listed above.

6. Which of the following represents the power series for the product of the two convergent power series

$$\frac{1}{1-2x} = 1 + 2x + 4x^2 + 8x^3 + \cdots$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots$$

A. $1 + 2x^2 - x^4 + \cdots$

B. $x - x^2 + 2x^3 + \cdots$

C. $x + \frac{3}{2}x^2 + \frac{10}{3}x^3 + \cdots$

D. $x - x^3 + \frac{4}{3}x^5 + \cdots$

E. $1 + x + 3x^2 + 5x^3 + \cdots$

7. To what simple function $f(x)$ does $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n\,2^n}$ converge near $x = 0$?

Hints: What is $f'(x)$? What is $f(0)$?

A. $\dfrac{1}{2}\ln|1-x|$

B. $\ln 2 - \ln|2-x|$

C. $\dfrac{x}{1-\dfrac{x}{2}}$

D. $\dfrac{1}{(1-\dfrac{x}{2})^2}$

E. $\dfrac{x}{(1-\dfrac{x}{2})^2}$

8. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$.

Hint: Let $f(x) = \displaystyle\sum_{n=0}^{\infty} x^n$, find $f'(x)$, and compare $f'(\frac{1}{2})$ to the given series.

A. 2

B. $\dfrac{1}{2}$

C. $\dfrac{\pi}{2}$

D. 1

E. 4

9. In the Taylor series representation of $f(x) = \ln x$ centered at $x = 1$, what is the coefficient of the term containing $(x-1)^3$?

A. 2

B. -2

C. $\dfrac{1}{3}$

D. $-\dfrac{1}{3}$

E. 1

10. The Maclaurin series for $\sin x$ is $\sin x = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$. If $f(x) = \displaystyle\int \sin(x^2)\, dx$, what is coefficient of the x^7 term the Maclaurin series for $f(x)$?

A. $\dfrac{1}{7!}$

B. $\dfrac{1}{7}$

C. 0

D. $-\dfrac{1}{42}$

E. $-\dfrac{1}{7!}$

11. The Maclaurin series for an unknown function $f(x)$ is $1 + \frac{2}{1!}x + \frac{3}{2!}x^2 + \frac{4}{3!}x^3 + \cdots$. What is the value of $f^{(99)}(0)$ (the 99th derivative at $x = 0$)?

A. 0
B. $\frac{99}{98!}$
C. 99
D. $\frac{100}{99!}$
E. 100

12. The binomial series is

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \ldots.$$

What is the power series representation of

$$f(x) = \frac{1}{\sqrt[4]{1+x}}?$$

A. $\sum_{n=0}^{\infty} \frac{(-1)^n[1 \cdot 5 \cdot 9 \cdot \cdots \cdot (4n+1)]}{4^n n!} x^n$
B. $\sum_{n=0}^{\infty} \frac{(-1)^n[1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n+1)]}{4^{n+1} n!} x^n$
C. $\sum_{n=0}^{\infty} \frac{(-1)^n[1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)]}{4^n n!} x^n$
D. $\sum_{n=0}^{\infty} \frac{(-1)^n(4n-3)}{4^n n!} x^n$
E. $\sum_{n=0}^{\infty} \frac{(-1)^n[1 \cdot 5 \cdot 9 \cdot \cdots \cdot (4n-3)]}{4^n n!} x^n$