MA 16200
Exam 3
04/11/2023
TEST/QUIZ NUMBER: 11

NAME ___________________ YOUR TA’S NAME ___________________

STUDENT ID # ______________ RECITATION TIME ______________

You must use a #2 pencil on the scantron answer sheet. Fill in the following on your scantron and blacken the bubbles

1. Your name. If there aren’t enough spaces for your name, fill in as much as you can.
2. Section number with a leading zero(s), e.g. 0023. (If you don’t know your section number, ask your TA.)
3. Test/Quiz number: 11
4. Student Identification Number: This is your Purdue ID number with two leading zeros
5. Blacken in your choice of the correct answer on the scantron answer sheet for questions 1–12.

There are 12 questions, each worth 8 points (you will earn 4 points for filling out your scantron correctly). Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

If you finish the exam before 8:50pm, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20pm. If you don’t finish before 8:50pm, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam booklet until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else’s test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, students must put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT SIGNATURE: ____________________________
1. Using the 2nd order Taylor polynomial of $f(x) = \ln(x)$ centered at $a = 3$,

$$ \ln (\pi) \approx \ln(3) + \frac{1}{3}(\pi - 3) - \frac{1}{18}(\pi - 3)^2. $$

According to the Taylor Remainder Theorem, this approximation has error equal to

A. $\frac{(\pi - 3)^3}{6c^3}$ for some $c$ between 3 and $\pi$
B. $\frac{(\pi - 3)^2}{2c^2}$ for some $c$ between 3 and $\pi$
C. $\frac{(\pi - 3)^2}{c^2}$ for some $c$ between 3 and $\pi$
D. $\frac{(\pi - 3)^3}{6c^2}$ for some $c$ between 3 and $\pi$
E. $\frac{(\pi - 3)^3}{3c^3}$ for some $c$ between 3 and $\pi$

2. Find all the values of $p$ such that the following series converges:

$$ \sum_{k=1}^{\infty} \sqrt{\frac{k^4 + 2k^2}{3 + k^p}} $$

A. $p > 2$
B. $p > 6$
C. $p > 8$
D. $p > 5$
E. $p > 4$
3. The series:

\[ \sum_{k=2}^{\infty} \frac{1}{k[\ln(k)]^2} \]

A. Converges by the Integral Test
B. Diverges by the Integral Test
C. Diverges by the Divergence Test
D. Converges by the Divergence Test
E. Diverges by comparison with \( \sum_{k=2}^{\infty} \frac{1}{k} \)

4. According to the root test, the series

\[ \sum_{k=1}^{\infty} \left[ \sin \left( \frac{1}{k} \right) \right]^k \]

A. Inconclusive because \( \rho = 0 \)
B. Converges because \( \rho < \infty \)
C. Diverges because \( \rho > 1 \)
D. Inconclusive because \( \rho = 1 \)
E. Converges because \( \rho < 1 \)
5. Which of the following statements are true?

I. If \( \lim_{n \to \infty} a_n = 0 \), then \( \sum_{n=1}^{\infty} a_n \) converges.

II. If \( \sum_{n=1}^{\infty} |a_n| \) converges, then \( \sum_{n=1}^{\infty} a_n \) converges.

III. If the sequence \( \{a_n\} \) is decreasing, then \( \sum_{n=1}^{\infty} (-1)^{n+1} a_n \) converges.

A. III only
B. II and III only
C. I and II only
D. I and III only
E. II only

6. Compute:

\[ \sum_{k=1}^{\infty} \frac{(-e)^{k+1}}{5^k} \]

A. \( \frac{5e}{5+e} \)
B. \( \frac{e}{5+e} \)
C. \( \frac{5}{5+e} \)
D. Series diverges
E. \( \frac{e^2}{5+e} \)
7. Which of the following series converge?

I. \[ \sum_{k=1}^{\infty} \frac{7^k}{k! + 10} \]

II. \[ \sum_{k=1}^{\infty} \frac{2 + (-1)^k}{2^k} \]

III. \[ \sum_{k=1}^{\infty} \sin\left(\frac{1}{\sqrt{k}}\right) \]

A. II and III only  
B. I and III only  
C. I, II, and III  
D. II only  
E. I and II only

8. For which of the following series is the ratio test inconclusive? (Only one is inconclusive)

A. \[ \sum_{k=1}^{\infty} \frac{k + 3}{5k - 1} \]

B. \[ \sum_{k=1}^{\infty} \frac{\sqrt{k}}{k!} \]

C. \[ \sum_{k=1}^{\infty} \frac{k!}{2^k} \]

D. \[ \sum_{k=1}^{\infty} \frac{(-10)^k}{k^{72k+1}} \]

E. \[ \sum_{k=1}^{\infty} \frac{e^k}{3k} \]
9. Determine the radius of convergence for the power series

\[ \sum_{k=1}^{\infty} \frac{k(x - 6)^k}{3^k} \]

A. 3  
B. \( \frac{1}{3} \)  
C. \( \infty \)  
D. 6  
E. \( \frac{1}{2} \)

10. Approximate \( e^{-0.1} \) using a 2nd-order Taylor polynomial centered at \( a = 0 \).

A. 0.895  
B. 0.91  
C. 0.89  
D. 0.915  
E. 0.905
11. Compute:
\[ \sum_{k=1}^{\infty} \ln(k) - \ln(k + 1) \]

A. \( \frac{1}{e} \)
B. 1
C. Series diverges
D. 0
E. \( \ln(2) \)

12. Let \( S = \sum_{k=1}^{\infty} \frac{(-1)^k}{(2k + 1)^2} \) Use the Alternating Series Estimation Theorem to find the smallest integer \( N \) such that we can be sure that \( |S_N - S| < \frac{1}{400} \), where \( S_N \) is the \( N \)th partial sum.

A. \( N = 12 \)
B. \( N = 8 \)
C. \( N = 9 \)
D. \( N = 11 \)
E. \( N = 10 \)
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