MA 16200: Third Midterm Examination

Spring 2026, Purdue University

Exam version: Ⓐ

Name: _____ PUID #: _____

Exam Instructions:

- Follow these instructions carefully. Failure to do so may result in your exam being invalidated and/or an academic integrity violation. All suspected violations of academic integrity will be reported to the Office of the Dean of Students.
- **VERY IMPORTANT:** Mark Ⓐ for Test Form on the top right corner of your Scantron sheet.
- Select your recitation section below. Write your name and PUID on the top of this cover page. **DO NOT WRITE ANYTHING ELSE** on the cover page.

	Sec	Time	TA Name
◯	0304	1:30PM	Fawzan Ali
◯	0303	2:30PM	Fawzan Ali
◯	0302	1:30PM	José Barrientos Lopez
◯	0301	2:30PM	José Barrientos Lopez
◯	0310	12:30PM	Dhruv Bhat
◯	0309	1:30PM	Dhruv Bhat
◯	0306	11:30AM	Rose Brickner
◯	0305	12:30PM	Rose Brickner
◯	0112	1:30PM	Trey Crouch
◯	0111	2:30PM	Trey Crouch
◯	0102	3:30PM	Luke Miga
◯	0101	4:30PM	Luke Miga

	Sec	Time	TA Name
◯	0308	11:30AM	Pedro Morales
◯	0307	12:30PM	Pedro Morales
◯	0107	3:30PM	Skip Moses
◯	0108	4:30PM	Skip Moses
◯	0311	8:30AM	Shivang Patel
◯	0312	9:30AM	Shivang Patel
◯	0110	3:30PM	Oliver Tan
◯	0109	4:30PM	Oliver Tan
◯	0103	12:30PM	Alex Yang
◯	0104	1:30PM	Alex Yang
◯	0106	3:30PM	Andy Yu
◯	0105	4:30PM	Andy Yu

- This exam consists of 12 questions for a total of 100 points.
- You have exactly one hour to complete the exam.
- Do not open the exam booklet or start writing before the proctor signals the start of the exam.
- Do not detach any page from the exam booklet.
- Additional pages for scratch work can be found every other page in the exam booklet.
- Calculators, electronic devices, books, or notes are **NOT ALLOWED**.
- Students may not look at anybody else's exam, and may not communicate with anybody else except with their TA or instructor if there is a question.
- If you finish the exam before 8:55 pm, you may leave the room after turning in the exam booklet. You may not leave the room before 8:20 pm. If you don't finish before 8:55 pm, **YOU MUST REMAIN SEATED** until your TA comes and collects your exam booklet. You must stop working when the proctor signals the end of exam.

Good luck!

Remember to mark all your answer choices on the Scantron sheet!

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1. (8 points) Which of the following series equals the repeating decimal

$$5.\overline{42} = 5.424242424242\ldots \quad ?$$

(1) $5 + \sum_{k=1}^{\infty} 42(0.01)^k$

(2) $5 + \sum_{k=1}^{\infty} (0.42)^k$

(3) $5 + \sum_{k=1}^{\infty} 42(0.99)^k$

(4) $5 + \sum_{k=1}^{\infty} 42(0.1)^k$

(5) $5 + \sum_{k=1}^{\infty} 99(0.42)^k$

2. (9 points) Given two arbitrary sequences $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$, determine which of the following three statements is/are always TRUE.

   (a) If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\{a_n\}_{n=1}^{\infty}$ is also convergent.

   (b) If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\sum_{n=162}^{\infty} a_n$ is also convergent.

   (c) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are both convergent, then $\{a_n/b_n\}_{n=1}^{\infty}$ is also convergent.

   Note: Partial credit is possible for this question.

(1) None of the above
(2) Only (a)
(3) Only (b)
(4) Only (c)
(5) Only (a) and (b)
(6) Only (a) and (c)
(7) Only (b) and (c)
(8) All of (a), (b), and (c)

Remember to mark all your answer choices on the Scantron sheet!

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Remember to mark all your answer choices on the Scantron sheet!

3. (8 points) Compute the third partial sum S_3 of the series

$$\sum_{n=1}^{\infty}(-1/2)^n$$

(1) $S_3 = -3/8$
(2) $S_3 = -1/3$
(3) $S_3 = -7/8$
(4) $S_3 = 3/4$
(5) $S_3 = 2/3$

4. (8 points) Find the sum of the series

$$\sum_{n=1}^{\infty}\frac{4}{(4n-3)(4n+1)}.$$

(1) 3/4
(2) 1
(3) 1/4
(4) 1/16
(5) 9/16

Remember to mark all your answer choices on the Scantron sheet!

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5. (8 points) Which one of the following conclusions is correct if the root test is applied to the series

$$\sum_{n=1}^{\infty} \left(\frac{n+2}{5n}\right)^{-n} \quad ?$$

(1) The series is absolutely convergent because the limit used in the root test is e^{-2}.

(2) The series is absolutely convergent because the limit used in the root test is $1/5$.

(3) The series is divergent because the limit used in the root test is 5.

(4) The series is divergent because the limit used in the root test is ∞.

(5) The root test is inconclusive because the limit used in the root test is 1.

6. (9 points) Which of the following convergence tests is/are applied correctly by themselves to determine that the series

$$\sum_{n=1}^{\infty} \frac{3^n}{2^n + n^2}$$

is divergent?

(a) The series is divergent by the direct comparison test with $\sum (3/2)^n$.

(b) The series is divergent by the divergence test.

(c) The series is divergent by the limit comparison test with $\sum 1^n$.

Note: Partial credit is possible for this question.

(1) None of the above

(2) Only (a)

(3) Only (b)

(4) Only (c)

(5) Only (a) and (b)

(6) Only (a) and (c)

(7) Only (b) and (c)

(8) All of (a), (b), and (c)

Remember to mark all your answer choices on the Scantron sheet!

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7. (8 points) What is the third-order Taylor polynomial for $f(x) = x^3 - 2x + 4$ centered at $x = 2$?

   (1) $8 + 10(x-2) + 12(x-2)^2 + 6(x-2)^3$
   (2) $4 + 10(x-2) + 12(x-2)^2 + 3(x-2)^3$
   (3) $8 + 10(x-2) + 6(x-2)^2 + (x-2)^3$
   (4) $8 + 10(x-2) + 6(x-2)^2 + 2(x-2)^3$
   (5) $8 + 10(x-2) + 12(x-2)^2 + (x-2)^3$

8. (8 points) What is the limit of the sequence that is defined by the recurrence relation

$$a_1 = 1; \quad a_n = \sqrt{2 + a_{n-1}} \quad ?$$

   (1) 1
   (2) $\sqrt{2}$
   (3) -1
   (4) 2
   (5) The sequence does not converge.

Remember to mark all your answer choices on the Scantron sheet!

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Remember to mark all your answer choices on the Scantron sheet!

9. (8 points) What are all values of p that can make the following series convergent?

$$\sum_{n=1}^{\infty} \sqrt[3]{\frac{5}{n^p+2}}$$

(1) $p \geq 0$
(2) $p > 3$
(3) $p > 1$
(4) $p \geq 3$
(5) $p \geq 1$

10. (10 points) Determine whether each one of the following series is absolutely convergent, conditionally convergent, or divergent.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(-2)^{2k+1}}$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$

Note: Partial credit is possible for this question.

(1) Series (a) is absolutely convergent; series (b) is absolutely convergent.
(2) Series (a) is absolutely convergent; series (b) is conditionally convergent.
(3) Series (a) is absolutely convergent; series (b) is divergent.
(4) Series (a) is conditionally convergent; series (b) is absolutely convergent.
(5) Series (a) is conditionally convergent; series (b) is conditionally convergent.
(6) Series (a) is conditionally convergent; series (b) is divergent.
(7) Series (a) is divergent; series (b) is absolutely convergent.
(8) Series (a) is divergent; series (b) is conditionally convergent.
(9) Series (a) is divergent; series (b) is divergent.

Remember to mark all your answer choices on the Scantron sheet!

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11. (8 points) If we use the second-order Taylor polynomial $p_2(x)$ for $f(x) = \cos(x)$ centered at $a = \pi/4$ to approximate the value of $\cos(\pi/3)$, we get

$$\cos\left(\frac{\pi}{3}\right) \approx \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(\frac{\pi}{12}\right) - \frac{\sqrt{2}}{4}\left(\frac{\pi}{12}\right)^2.$$

By Taylor's remainder theorem, what can we say about the error of this approximation?

(1) The error equals $\dfrac{\cos(c)}{6}\left(\dfrac{\pi}{12}\right)^3$ for some c between $\dfrac{\pi}{4}$ and $\dfrac{\pi}{3}$.

(2) The error equals $\dfrac{\cos(c)}{12}\left(\dfrac{\pi}{12}\right)^3$ for some c between $\dfrac{\pi}{4}$ and $\dfrac{\pi}{3}$.

(3) The error equals $\dfrac{\cos(c)}{9}\left(\dfrac{\pi}{12}\right)^3$ for some c between $\dfrac{\pi}{4}$ and $\dfrac{\pi}{3}$.

(4) The error equals $\dfrac{\sin(c)}{12}\left(\dfrac{\pi}{12}\right)^3$ for some c between $\dfrac{\pi}{4}$ and $\dfrac{\pi}{3}$.

(5) The error equals $\dfrac{\sin(c)}{6}\left(\dfrac{\pi}{12}\right)^3$ for some c between $\dfrac{\pi}{4}$ and $\dfrac{\pi}{3}$.

12. (8 points) Consider the series

$$S = \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2+3}$$

By the error estimate for alternating series, how many terms are needed for the n-th partial sum S_n to be within 10^{-2} of the true sum S?

(1) $n = 9$

(2) $n = 4$

(3) $n = 19$

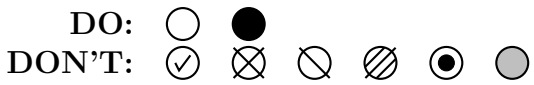
(4) $n = 14$

(5) $n = 99$

Remember to mark all your answer choices on the Scantron sheet!

Scantron Instructions:

- Fill the circles of your choice completely with a **#2 PENCIL**. If you need to change your choice, erase the mark completely.



- Write down the following information on the top of your Scantron sheet.
  - INSTRUCTOR: Write down the name of your teaching assistant, which can be found on the front cover.
  - COURSE: Write down "MA 162."
  - DATE: Write down today's date.
  - TEST: Don't worry about this.
  - SIGNATURE: Put down your signature.
- **VERY IMPORTANT:** Mark Ⓐ for Test Form.
- Write down your name, and fill in the corresponding bubbles.
  - If your name does not fit in the provided space, fill in as much as possible.
  - If your name contains a diacritic (accent), fill in the corresponding letter without the accent mark. For example, fill in `PADME` for Padmé.
  - If your name contains hyphen(s), apostrophe(s), or other special characters, omit the special character and run the letters together. For example, fill in `OBIWAN` for Obi-Wan.
- Fill in the section number of your **RECITATION**, which always starts with a leading zero. This information can be found on the front cover.
- Fill in your PUID number, which always starts with two leading zeros. For example, if your student ID is 12345678 or 012345678, you should fill in `0012345678` for Student ID.
- Mark **EXACTLY ONE** answer for each question. Questions with more than one marked circle will receive no credit.
- Only what you marked on the Scantron sheet will be graded for score.
- Partial credit will not be awarded, unless otherwise indicated by individual questions.

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