MATH 162 – FALL 2006 – FINAL EXAM
DECEMBER 13, 2006

STUDENT NAME & ID————————————————————-

RECITATION TIME & INSTRUCTOR——————————————————

INSTRUCTIONS

1. Verify that you have **12 pages.**
2. Fill in the blank spaces above.
3. Use a number 2 pencil to write on your **mark-sense sheet**.
4. **On your mark sense sheet**, write your name, your student ID number, the division and section numbers of your recitation, and fill the corresponding circles.
5. Mark the letter of your response for each question on the **mark-sense sheet.**
6. There are 22 questions, **the first two are worth 10 points each. The others are worth 9 points each.**
7. Show as much as possible of your work. Although this exam will be machine graded, in certain situations it may be necessary that we look at your exam.
8. No books, notes or calculators may be used.

USEFUL FORMULAS

$$\sin^2 x = \frac{1-\cos(2x)}{2}, \quad \cos^2 x = \frac{1+\cos(2x)}{2}, \quad 1+\tan^2 x = \sec^2 x$$

$$\cos(\frac{\pi}{3}) = \sin(\frac{\pi}{6}) = \frac{1}{2}, \quad \cos(\frac{\pi}{6}) = \sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}, \quad \cos(\frac{\pi}{4}) = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$$

$$\text{Volumes} \quad V = \pi\int_a^b A(x)\,dx, \quad V = \int_a^b 2\pi x f(x)\,dx$$

Moments and center of mass

$$M_x = \int_a^b \frac{1}{2}\left((f(x))^2 - (g(x))^2\right)dx, \quad M_y = \int_a^b x\,(f(x)-g(x))\,dx, \quad \overline{x} = \frac{M_y}{M}, \quad \overline{y} = \frac{M_x}{M},$$

$$\text{Arc length} \quad L = \int_a^b \sqrt{1+(f'(x))^2}\;\,dx$$

$$\text{Area of a surface of revolution about the } x-\text{axis} \quad S = \int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$$

1) (10 points) The volume of the parallelepiped determined by the vectors $\vec{v_1} = \vec{i} + 2\vec{j} + 3\vec{k}$, $\vec{v_2} = -\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{v_3} = \vec{j} + 4\vec{k}$ is equal to

A) 7

B) 6

C) 5

D) 8

E) 9

2) (10 points) The area of the triangle with vertices $P_1(1, 1, 3)$, $P_2(0, 1, 4)$ and $P_3(-1, 2, 1)$ is equal to

A) $\sqrt{3}$

B) 18

C) $3\sqrt{2}$

D) $\frac{3\sqrt{2}}{2}$

E) 9

3) (9 points) The area of the region bounded by the curves $x = y^2 - 4y$ and $x = 2y - y^2$ is equal to

A) 10

B) 9

C) 7

D) 8

E) 11

4) (9 points) The region in the first quadrant bounded by the curves $y = x^2$ and $x = y$ is rotated about the axis $x = 0$. The volume of the resulting solid is equal to

A) $\frac{2\pi}{15}$

B) $\frac{\pi}{3}$

C) $\frac{\pi}{6}$

D) $\frac{\pi}{4}$

E) 2π

5) (9 points) The moment about the y-axis of the region in the first quadrant bounded by $y = x^2$ and $y = x - x^2$, and density of mass equal to one is

A) $\frac{1}{72}$

B) $\frac{1}{96}$

C) $\frac{1}{64}$

D) $\frac{1}{84}$

E) $\frac{1}{102}$

6) (9 points) Find the area of the surface of the solid obtained by rotating the curve $y = \frac{1}{3}x^3$, with $0 \leq x \leq 1$, about the x-axis

A) $\frac{\pi}{9}(2\sqrt{3} - 1)$

B) $\frac{\pi}{6}(2\sqrt{3} - 1)$

C) $\frac{\pi}{9}(2\sqrt{2} - 1)$

D) $\frac{\pi}{6}(2\sqrt{2} - 1)$

E) $\frac{\pi}{9}$

7) (9 points) The integral

$$\int_0^{\frac{\pi}{2}} x \cos x \; dx \quad \text{is equal to}$$

A) $\frac{\pi}{2} - 1$

B) $\frac{\pi}{2}$

C) $\frac{\pi}{4}$

D) $\frac{3\pi}{2}$

E) $\frac{3\pi}{2} - 1$

8) (9 points) The integral

$$\int_0^{\frac{\pi}{3}} (\tan x)^3 \sec x \; dx \quad \text{is equal to}$$

A) $\frac{4}{3}$

B) $\frac{2}{3}$

C) $\sqrt{3}$

D) $2\sqrt{3}$

E) $\frac{1}{7}$

9) (9 points) Which substitution should be used to evaluate

$$\int \frac{x}{\sqrt{8-7x^2}}\,dx?$$

A) $x = \sqrt{\frac{8}{7}}\sin\theta$

B) $x = \sqrt{\frac{3}{5}}\sec\theta$

C) $x = \sqrt{\frac{7}{8}}\cos\theta$

D) $x = \frac{8}{7}\tan\theta$

E) $x = \frac{7}{8}\sec\theta\tan\theta$

10) (9 points) The integral

$$\int_1^\infty \frac{dx}{x^2\sqrt{1+x^2}} \quad \text{is equal to}$$

A) $\sqrt{2}$

B) $2\sqrt{2}$

C) $\sqrt{2}+1$

D) $\sqrt{2}-1$

E) $2\sqrt{2}-1$

11) (9 points) The integral

$$\int_2^3 \frac{x+1}{x(x-1)}\, dx \text{ is equal to}$$

A) $\ln\left(\frac{8}{3}\right)$

B) $\ln\left(\frac{5}{3}\right)$

C) $\ln\left(\frac{7}{4}\right)$

D) $\ln\left(\frac{9}{5}\right)$

E) $\ln\left(\frac{10}{6}\right)$

12)(9 points) The length of the curve $y = \frac{1}{2}x^2 - \frac{1}{4}\ln x$, $1 \leq x \leq 2$ is

A) $\frac{3}{2} + \frac{1}{4}\ln 2$

B) $\frac{1}{2} + \frac{1}{3}\ln 2$

C) $2 + \frac{1}{8}\ln 2$

D) $4 + 2\ln 2$

E) $1 + \ln 2$

13) (9 points) Let $f(x)$ be a function defined for $x \geq 1$, such that $1 \geq f(x) \geq \frac{1}{\sqrt{x}}$, for all $x \geq 1$. What can be said about the series

$$S_1 = \sum_{n=1}^{\infty} \frac{f(n)}{\sqrt{n}}, \quad S_2 = \sum_{n=1}^{\infty} \frac{f(n)}{n^2} \quad ?$$

A) S_1 and S_2 converge

B) S_1 diverges and S_2 converges

C) S_1 converges and S_2 diverges

D) S_1 and S_2 diverge

E) S_2 converges, but S_1 might converge or diverge.

14) (9 points) Using that $\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$, and the limit comparison theorem, the following is true about the series

$$S_1 = \sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n}\right), \quad S_2 = \sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n^2}\right):$$

A) S_1 and S_2 converge

B) S_1 diverges and S_2 converges

C) S_1 converges and S_2 diverges

D) S_1 and S_2 diverge

E) S_2 converges, but S_1 might converge or diverge.

15)(9 points) Find the smallest number of terms which one needs to add to find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!\, n}$ with an error strictly less than 10^{-4}.

A) 4 terms

B) 5 terms

C) 6 terms

D) 7 terms

E) 8 terms

16)(9 points) Which of the following is the interval of convergence of the power series

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^4(x-1)^n}{3^n(n^5+2)} \ ?$$

A) $(2, 4)$

B) $[0, 3)$

C) $(-2, 4]$

D) $[0, 3)$

E) $[-2, 4)$

17)(9 points) If the Maclaurin series of a function $f(x)$ is

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{3n^2(n+5)}$$

then $f^{(5)}(0)$ is equal to

A) $4/25$

B) $-15/6$

C) $8/25$

D) $9/7$

E) $8/5$

18) (9 points) Knowing that $\frac{d^k}{dx^k}(1-x)^{-1} = k!(1-x)^{-k-1}$, we can say that the Maclaurin series of $f(x) = \frac{1}{(1-x)^4}$ is given by

A) $\sum_{n=3}^{\infty}(-1)^n \frac{n(n-1)(n-2)}{6} \quad x^{n-3}$

B)$\sum_{n=3}^{\infty} \frac{n(n-1)(n-2)}{6} \quad x^{n-3}$

C) $\sum_{n=2}^{\infty}(-1)^n n(n-1)x^{n-2}$

D) $\sum_{n=2}^{\infty} \frac{x^{n-2}}{n(n-1)}$

E) $\sum_{n=2}^{\infty} \frac{x^{n-2}}{2n(n-1)}$

19) (9 points) Find the sum of the series

$$\sum_{n=3}^{\infty}\left(\frac{1}{n+2}-\frac{1}{n+3}\right)$$

A) $\frac{1}{2}$

B) $\frac{1}{3}$

C) $\frac{1}{4}$

D) 1

E) $\frac{1}{5}$

20) (9 points) Find the sum of the series

$$\sum_{n=4}^{\infty}\frac{(-4)^{n+3}}{3^{n+1}}$$

A) 81/112

B) 27/64

C) 81/20

D) 9/32

E) it diverges

21) (9 points) The line tangent to the curve given by $x(t) = t^2 + t + 1$, $y(t) = t^3 + t + 8$ at $(1, 8)$ can be represented by

A) $y = 2x + 6$

B) $y = 3x + 5$

C) $y = x + 7$

D) $y = 9 - x$

E) $y = 10 - 2x$

22) (9 points) Find the area of the region bounded by the curve $r = \sin 4\theta$ for $0 \leq \theta \leq \frac{\pi}{4}$.

A) $\frac{\pi}{16}$

B) $\frac{\pi}{8}$

C) $\frac{3\pi}{4}$

D) $\frac{4\pi}{3}$

E) $\frac{2\pi}{5}$