MA 162 FINAL EXAM - Version 01 Fall 2013

IT IS YOUR JOB TO FILL IN THE SCANTRON WITH CORRECT EXAM VERSION

NAME ______________________________

PUID# ______________________________

RECITATION INSTRUCTOR ______________________________

RECITATION TIME ______________________________

LECTURER ______________________________

INSTRUCTIONS

1. Fill in all the information above and fill in the version number of the test on your scantron.
2. Using a #2 pencil, write each of the following items and fill in the corresponding circles on your scantron:
   (a) Print your name (last name, first name).
   (b) Under SECTION NUMBER, write in your **4** digit section number (for example 0012 or 0003).
   (c) Under PUID, write in your Purdue student I.D. number.
3. Make sure you have a complete test. There are **25** problems, each worth 8 points.
4. Do any necessary work for each problem on the pages of this booklet. Circle your answers in this booklet in case of a lost scantron.
5. NO books, notes or calculators are allowed on any exam. Turn off all electronic devices.
6. After you have finished the test, turn in your scantron and this test booklet to your recitation instructor.

Brief Table of Maclaurin Series

$$\frac{1}{1-x}=\sum_{n=0}^{\infty} x^n\,, \quad \cos x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}\,, \quad \sin x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} \quad e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}$$

$$(1+x)^k=1+kx+\frac{k(k-1)}{2!}x^2+\frac{k(k-1)(k-2)}{3!}x^3+\cdots$$

1. Find a vector $\vec{\mathbf{u}}$ that is perpendicular to the plane which contains $P(-1,-1,1)$, $Q(1,2,3)$ and $R(3,2,-1)$.

A. $\langle 2,4,-1\rangle$

B. $\langle 2,-2,1\rangle$

C. $\langle 2,-1,3\rangle$

D. $\langle -1,1,3\rangle$

E. $\langle -2,1,1\rangle$

2. Find the value of x such that the two vectors $\langle 2x,1,2\rangle$ and $\langle 1,3x,-5\rangle$ are perpendicular to each other.

A. 3

B. 2

C. 4

D. 1/3

E. 1/4

3. Find the area of the triangle with vertices $(1, 0, 2)$, $(3, 2, 1)$ and $(2, 1, 3)$.

A. $\frac{3\sqrt{2}}{2}$

B. $\frac{5}{2}$

C. $2\sqrt{5}$

D. $3\sqrt{3}$

E. $\frac{3}{4}$

4. Find the area between the curves $y = 3x - x^2$ and $y = x$.

A. $\frac{5}{6}$

B. $\frac{4}{3}$

C. $\frac{2}{3}$

D. $\frac{7}{6}$

E. 1

5. The region in the first quadrant bounded by the y-axis, the graph of $x + y = 2$ and the graph of $x - y = 0$ is revolved about the x-axis. Find the volume V of the solid generated.

A. $\frac{3\pi}{5}$

B. $\frac{\pi}{2}$

C. 2π

D. $\frac{14\pi}{3}$

E. π

6. The region bounded by $y = 2x$, $x = 2$ and the x-axis is rotated about the line $x = 3$. The volume of the resulting solid is given by (use the cylindrical shell method)

A. $\int_0^2 2\pi(3 - x)(2x - 3)dx$

B. $\int_0^2 2\pi(3 - 2x)(x)dx$

C. $\int_0^2 2\pi(3 - 2x)dx$

D. $\int_0^2 2\pi(3 - x)(2x - 2)dx$

E. $\int_0^2 2\pi(3 - x)(2x)dx$

7. Evaluate $\int_1^e x^2 \ln x dx$.

A. $\dfrac{e^3+1}{3}$

B. $\dfrac{e^3}{3}$

C. $\dfrac{e^2+e}{9}$

D. $\dfrac{2e^3}{9}$

E. $\dfrac{2e^3+1}{9}$

8. An appropriate trig substitution will convert the definite integral $\int_1^3 \sqrt{x^2+2x-3}\,dx$ into which of the following definite integrals?

A. $\int_0^{\pi/3} 4\sec\theta\tan\theta d\theta$

B. $\int_0^{\pi/3} 4\sec\theta\tan^2\theta d\theta$

C. $\int_0^{\pi/3} 2\sec\theta\tan^3\theta d\theta$

D. $\int_0^{\pi/6} 2\sec\theta\tan^2\theta d\theta$

E. $\int_0^{\pi/3} \sec\theta\tan\theta d\theta$

9. Compute the length of the curve $y = \frac{x^3}{3} + \frac{1}{4x}$ for $1 \leq x \leq 2$.

A. $\frac{53}{24}$

B. $\frac{49}{24}$

C. $\frac{59}{24}$

D. $\frac{55}{24}$

E. $\frac{58}{24}$

10. A force of 10 lbs is required to stretch a spring 1 foot beyond its natural length. How much work is required to stretch the spring from 1 foot beyond its natural length to 2 feet beyond its natural length?

A. $\frac{19}{2}$ ft-lbs

B. 20 ft-lbs

C. 5 ft-lbs

D. 15 ft-lbs

E. $\frac{3}{2}$ ft-lbs

11. Let D be the region in the plane bounded by $y = x^2$, $y = -x$ and satisfying $0 \leq x \leq 2$. If the centroid of D is $(\bar{x}, \bar{y})$, compute $\bar{x}$.

A. $\frac{9}{4}$

B. $\frac{20}{3}$

C. $\frac{14}{3}$

D. $\frac{10}{7}$

E. $\frac{8}{5}$

12. Evaluate $\int_0^{\frac{3}{2\sqrt{2}}} \frac{dx}{\sqrt{9 - 4x^2}}$.

A. $\frac{\pi}{8}$

B. $\frac{\pi}{4\sqrt{3}}$

C. $\frac{\pi}{12}$

D. $\frac{\pi}{6}$

E. $\frac{\pi}{8\sqrt{3}}$

13. Which of the following improper integrals will diverge?

(I) $\displaystyle\int_0^1 \frac{1}{x^{\frac{1}{2}}}\,dx$ (II) $\displaystyle\int_1^\infty \frac{1}{x^{\frac{1}{2}}}\,dx$ (III) $\displaystyle\int_1^\infty \frac{1}{1+x^2}\,dx$

A. only (I)

B. only (II)

C. (I) and (II)

D. (II) and (III)

E. (I) and (III)

14. Which of the following sequences will converge?

(i) $\left\{\dfrac{n+1}{n-1}\right\}$ (ii) $\left\{(-1)^n + \dfrac{1}{n}\right\}$ (iii) $\{n(n-1)\}$ (iv) $\left\{\dfrac{n!}{(n+1)!}\right\}$ (v) $\left\{\dfrac{n!}{2^n}\right\}$

A. all diverge

B. only (ii) and (v)

C. (i), (iv) and (v)

D. only (i) and (iv)

E. (iv) and (v)

15. Compute $\displaystyle\sum_{n=1}^{\infty} \frac{3 \cdot 2^n + (-3)^n}{4^n}$

A. $\dfrac{15}{4}$

B. $\dfrac{24}{7}$

C. $\dfrac{9}{14}$

D. $\dfrac{22}{7}$

E. $\dfrac{18}{7}$

16. Which one of the following series is conditionally convergent?

A. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

B. $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{(-1)^n}{n}\right)$

C. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+n}}$

D. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{e^n}{\sqrt{n}}$

E. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

17. The set of all x for which the power series $\sum_{n=1}^{\infty} \frac{2^n}{\sqrt{n}} x^n$ converges is

A. $-\frac{1}{2} \leq x < \frac{1}{2}$

B. $-\frac{1}{2} < x < \frac{1}{2}$

C. $-\frac{1}{2} \leq x \leq \frac{1}{2}$

D. $x = 0$ only

E. $-\infty < x < \infty$

18. The Maclaurin series for the function $f(x) = \frac{x}{(1+x^2)^2}$ is

A. $\sum_{n=1}^{\infty}(-1)^n x^{2n}$

B. $\sum_{n=1}^{\infty}(-1)^n 2n x^{2n-1}$

C. $\sum_{n=1}^{\infty}(-1)^n n x^{2n-1}$

D. $\sum_{n=1}^{\infty}(-1)^{n-1} n x^{2n-1}$

E. $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$

19. Let $S = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3 - n^2 + 1}$ and let $S_N = \sum_{n=1}^{N} \frac{(-1)^n}{n^3 - n^2 + 1}$.

What is the smallest N such that the Alternating Series Estimation Test implies that $|S_N - S| < .01$?

A. 9

B. 10

C. 4

D. 5

E. 11

20. Which of the series below converge?

(I) $\sum_{n=1}^{\infty} \left(\frac{n-1}{2n+1}\right)^n$ (II) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{1+\ln n}$ (III) $\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{2n^4} - n}$

A. (I) and (II)

B. (I)

C. (I), (II), (III)

D. (II)

E. (II), (III)

21. Suppose $f(x) = x^3e^x$. Find $f^{(6)}(0)$.

A. 6!

B. 24

C. 6

D. 48

E. 120

22. Let $x = t^2$, $y = t^2 + t$.

Find $\dfrac{d^2y}{dx^2}$ at the point $(1, 2)$.

A. $1/2$

B. $-1/2$

C. $-1/4$

D. $1/4$

E. 2

23. The equation $4\cos\theta + \sin\theta = \dfrac{2}{r}$ in polar coordinates represents part of

A. a straight line

B. a circle

C. a parabola

D. an ellipse which is not a circle

E. a cycloid

24. Find the foci of the ellipse

$$4(x-1)^2 + (y-2)^2 = 9$$

A. $\left(1 \pm \dfrac{3\sqrt{3}}{2}, 2\right)$

B. $(1 \pm 3, 2)$

C. $(1, 2 \pm 3)$

D. $\left(1, 2 \pm \dfrac{3\sqrt{3}}{2}\right)$

E. $\left(1, 2 \pm \dfrac{3}{2}\right)$

25. If $z = 4 + 4i$ and $w = 1 + \sqrt{3}i$, express the product zw in polar form.

A. $8\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

B. $8\sqrt{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$

C. $8\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

D. $8\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$

E. $4\sqrt{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$