Name ______________________ 10–digit PUID ______________________

Recitation Instructor ______________________ Recitation Section Number ____________

Lecture's Name ______________________

Instructions:

1. Academic Integrity
   a. Students may not open the exam until instructed to do so.
   b. Students must obey the orders and requests by all proctors, TAs, and lecturers.
   c. No student may leave in the first 20 minutes or in the last 10 minutes of the exam.
   d. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody elses test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
   e. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
   f. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

   I have read and understand the exam rules stated above:

   STUDENT NAME (print): ______________________

   STUDENT SIGNATURE: ______________________

2. Fill in all the information requested above. On the scantron sheet fill in your name, student ID number, and the section number of your recitation with an extra 0 at the left. Blacken the correct circles.
3. This booklet contains 25 problems worth 8 points each. The maximum score is 200 points.
4. For each problem mark your answer on the scantron sheet and also circle it in this booklet.
5. Work only on the pages of this booklet.

Mark **TEST 01** on your scantron!

1. Find the cosine of the angle θ between $\vec{v} = \langle 2, -1, 2 \rangle$ and $\vec{w} = \langle 4, 1, 1 \rangle$.

A. $\dfrac{7}{\sqrt{20}}$

B. $\dfrac{1}{\sqrt{2}}$

C. $\dfrac{5}{9\sqrt{2}}$

D. $\dfrac{7}{9\sqrt{2}}$

E. $\dfrac{8}{15\sqrt{3}}$

2. Find the area of the triangle with vertices at $(1, 0, 0)$, $(3, 1, 1)$, and $(4, 1, 2)$.

A. $\dfrac{\sqrt{5}}{2}$

B. $\dfrac{\sqrt{3}}{2}$

C. $\sqrt{2}$

D. $\dfrac{\sqrt{6}}{2}$

E. $\dfrac{\sqrt{19}}{2}$

3. Find the area of the region D in the plane bounded by $x = 2y - 2y^2$ and $x = y^2 - 4y$.

A. $\frac{16}{3}$

B. $\frac{35}{9}$

C. 5

D. $\frac{16}{3}$

E. 4

4. Let D be the region in the first quadrant bounded by $y = 2 - x^2$, $y = x^2$ and $x = 0$. Find the volume of the solid obtained by rotating D about the x–axis.

A. $\frac{38\pi}{15}$

B. $\frac{32\pi}{15}$

C. $\frac{8\pi}{3}$

D. $\frac{13\pi}{5}$

E. $\frac{7\pi}{3}$

5. The region bounded by $y = 2x$, $x = 2$ and the x-axis is rotated about the line $x = 3$. The volume of the resulting solid is given by (use the cylindrical shell method)

   A. $\int_0^2 2\pi(3-x)(2x-3)dx$

   B. $\int_0^2 2\pi(3-2x)(x)dx$

   C. $\int_0^2 2\pi(3-2x)dx$

   D. $\int_0^2 2\pi(3-x)(2x-2)dx$

   E. $\int_0^2 2\pi(3-x)(2x)dx$

6. A conical tank with height 2 ft. and radius at the top of 1/2 ft. is full of water (62.5 lbs/ft^3). How much work (in ft-lb) is needed to pump all the water to the top of the tank?

   A. $62.5\pi\left(\frac{1}{24}\right)$

   B. $62.5\pi\left(\frac{1}{8}\right)$

   C. $62.5\pi\left(\frac{1}{16}\right)$

   D. $62.5\pi\left(\frac{1}{12}\right)$

   E. $62.5\pi\left(\frac{1}{4}\right)$

7. $\int_1^e x^2 \ln x dx$.

A. $\frac{e^3+1}{3}$

B. $\frac{e^3}{3}$

C. $\frac{e^2+e}{9}$

D. $\frac{2e^3}{9}$

E. $\frac{2e^3+1}{9}$

8. Compute $\int_0^{\frac{\pi}{3}} \tan^3 x dx$.

A. $3 - \frac{\sqrt{3}}{2}$

B. $4\sqrt{3} + \ln 2$

C. $\frac{\sqrt{3}}{2} - \ln 2$

D. $2\sqrt{3} - \frac{1}{2}$

E. $\frac{3}{2} - \ln 2$

9. An appropriate trig substitution will convert the definite integral $\int_1^3 \sqrt{x^2+2x-3}\,dx$ into which of the following definite integrals?

A. $\int_0^{\pi/3} 4\sec\theta\tan\theta d\theta$

B. $\int_0^{\pi/3} 4\sec\theta\tan^2\theta d\theta$

C. $\int_0^{\pi/3} 2\sec\theta\tan^3\theta d\theta$

D. $\int_0^{\pi/6} 2\sec\theta\tan^2\theta d\theta$

E. $\int_0^{\pi/3} \sec\theta\tan\theta d\theta$

10. Compute $\int_1^2 \frac{4\ dx}{x(x^2+4)}$

A. $\ln 2 - \frac{1}{2}\ln 5$

B. $\frac{1}{2}\ln 2 + \ln 5$

C. $\frac{1}{2}(\ln 5 - \ln 2)$

D. $\ln 2 + \ln 5$

E. $\frac{1}{2}\ln 2 - \ln 5$

11. Compute $\int_0^2 \frac{dx}{\sqrt{4-x^2}}$

A. $\frac{\pi}{4}$

B. $\frac{3}{4}$

C. 2π

D. $\frac{\pi}{2}$

E. Integral Diverges

12. Compute the length of

$$y = \frac{x^3}{3} + \frac{1}{4x}, \quad 1 \leq x \leq 2.$$

A. $\frac{53}{24}$

B. $\frac{59}{24}$

C. $\frac{29}{12}$

D. $\frac{31}{12}$

E. $\frac{55}{24}$

13. Which of the following sequences will converge?

(i) $\left\{\frac{n+1}{n-1}\right\}$ (ii) $\left\{(-1)^n + \frac{1}{n}\right\}$ (iii) $\{n(n-1)\}$ (iv) $\left\{\frac{n!}{(n+1)!}\right\}$ (v) $\left\{\frac{n!}{2^n}\right\}$

A. all diverge

B. only (ii) and (v)

C. (i), (iv) and (v)

D. only (i) and (iv)

E. only (iv) and (v)

14. $\sum_{n=2}^{\infty} \left(\frac{2}{7}\right)^n =$

A. $\frac{7}{5}$

B. $\frac{2}{5}$

C. $\frac{4}{35}$

D. $\frac{5}{2}$

E. $\frac{5}{7}$

15. Given the series

$$\text{I.} \sum_{n=1}^{\infty} \frac{1}{\sqrt{2n+1}} \quad \text{and} \quad \text{II.} \sum_{n=1}^{\infty} \frac{\ln n}{n},$$

A. Both I. and II. converge.

B. Both I. and II. diverge.

C. I. converges and II. diverges.

D. I. diverges and II. converges.

16. For the series $\sum_{n=0}^{\infty}(-1)^n \frac{n}{2^n}$ find the smallest integer N such that $\sum_{n=0}^{N}(-1)^n \frac{n}{2^n}$ approximates the sum of the series with error less than 0.1.

A. $N = 1$

B. $N = 2$

C. $N = 5$

D. $N = 6$

E. $N = 8$

17. Which one of the following series is conditionally convergent?

A. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

B. $\sum_{n=1}^{\infty} \left(1 + \frac{(-1)^n}{n}\right)$

C. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+n}}$

D. $\sum_{n=1}^{\infty} (-1)^n \frac{e^n}{\sqrt{n}}$

E. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$.

18. The set of all x for which the power series $\sum_{n=1}^{\infty} \frac{2^n}{\sqrt{n}} x^n$ converges is

A. $-\frac{1}{2} \leq x < \frac{1}{2}$

B. $-\frac{1}{2} < x < \frac{1}{2}$

C. $-\frac{1}{2} \leq x \leq \frac{1}{2}$

D. $x = 0$ only

E. $-\infty < x < \infty$

19. The Maclaurin series for the function $f(x) = \dfrac{x}{(1+x^2)^2}$ is

A. $\sum_{n=1}^{\infty}(-1)^n x^{2n}$

B. $\sum_{n=1}^{\infty}(-1)^n 2nx^{2n-1}$

C. $\sum_{n=1}^{\infty}(-1)^n nx^{2n-1}$

D. $\sum_{n=1}^{\infty}(-1)^{n-1} nx^{2n-1}$

E. $\sum_{n=1}^{\infty}\dfrac{(-1)^n}{2n+1}x^{2n+1}$

20. Let $f(x) = \sum_{n=3}^{\infty}\dfrac{n+2}{(n-2)!}(x-2)^n$. Then $f^{(162)}(2)$ is

A. $(164)(162)(161)$

B. $\dfrac{(x-2)^{162}}{162!}$

C. $\dfrac{164}{160!}$

D. 0

E. $\dfrac{1}{(162)(161)}$

21. If one uses the first three terms of the (binomial) series for $(1+x)^{1/3}$ to find an approximation for $\sqrt[3]{1.1}$, the result is

A. $1+\left(\frac{1}{2}\right)\left(\frac{1}{10}\right)+\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)\left(\frac{1}{100}\right)$

B. $1+\left(\frac{1}{3}\right)\left(\frac{1}{10}\right)+\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)\left(\frac{1}{100}\right)$

C. $1+\left(\frac{1}{2}\right)\left(\frac{1}{10}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(\frac{1}{100}\right)$

D. $1+\left(\frac{1}{3}\right)\left(\frac{1}{10}\right)+\left(\frac{1}{2}\right)\left(\frac{-2}{3}\right)\left(\frac{1}{100}\right)$

E. $1+\left(\frac{1}{3}\right)\left(\frac{1}{10}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)\left(\frac{1}{100}\right)$

22. Let $x=t^2$, $y=t^2+t$. Find $\frac{d^2y}{dx^2}$ at the point $(1,2)$

A. $1/2$

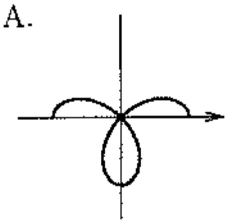
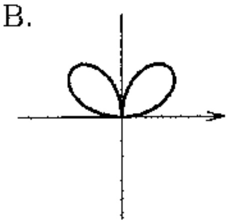
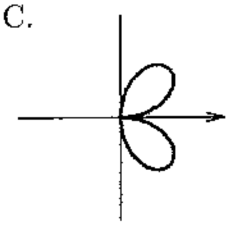
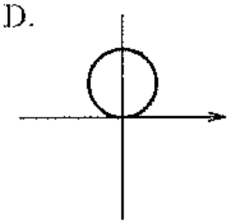
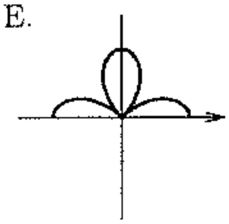
B. $-1/2$

C. $-1/4$

D. $1/4$

E. 2

23. The graph of $r = \cos 2\theta,\ 0 \leq \theta \leq \pi$ looks most like

A. 
B. 
C. 
D. 
E. 

24. An equation for the parabola with vertex $(1, -2)$ and directrix $x = 5$ is

A. $(y+2)^2 = -16(x-1)$

B. $(y+2)^2 = 16(x-1)$

C. $y+2 = -16(x-1)^2$

D. $y+2 = 16(x-1)^2$

E. $(y-2)^2 = 16(x+1)$

25. Write the complex number $-2+2\sqrt{3}i$ in polar form with argument θ between 0 and 2π.

A. $4\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)$

B. $4\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$

C. $4\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$

D. $2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$

E. $2\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$