# MA 16200: Final Examiniation

**Fall 2025, Purdue University**

Exam version: 01

Name: _____ PUID #: _____

**Exam Instructions:**

- Follow these instructions carefully. Failure to do so may result in your exam being invalidated and/or an academic integrity violation. All suspected violations of academic integrity will be reported to the Office of the Dean of Students.
- Mark the circle of your recitation section below.

| | Sec | Time | TA Name |
|---|---|---|---|
| ◯ | 121 | 7:30AM | Jonah Boan |
| ◯ | 109 | 7:30AM | Juliet Raginsky |
| ◯ | 202 | 7:30AM | Dhruv Wadhwa |
| ◯ | 123 | 8:30AM | Jonah Boan |
| ◯ | 111 | 8:30AM | Juliet Raginsky |
| ◯ | 204 | 8:30AM | Dhruv Wadhwa |
| ◯ | 902 | 9:30AM | Luke Miga |
| ◯ | 212 | 9:30AM | Kathryn Moran |
| ◯ | 904 | 10:30AM | Luke Miga |
| ◯ | 210 | 10:30AM | Kathryn Moran |
| ◯ | 115 | 11:30AM | Skip Moses |
| ◯ | 218 | 11:30AM | Uyen Nguyen |
| ◯ | 909 | 11:30AM | Shivang Patel |
| ◯ | 113 | 12:30PM | Skip Moses |
| ◯ | 220 | 12:30PM | Uyen Nguyen |

| | Sec | Time | TA Name |
|---|---|---|---|
| ◯ | 910 | 12:30PM | Shivang Patel |
| ◯ | 117 | 12:30PM | Alex Yang |
| ◯ | 214 | 1:30PM | Fawzan Ali |
| ◯ | 208 | 1:30PM | Oliver Tan |
| ◯ | 119 | 1:30PM | Alex Yang |
| ◯ | 216 | 2:30PM | Fawzan Ali |
| ◯ | 206 | 2:30PM | Oliver Tan |
| ◯ | 224 | 3:30PM | Ayo Aitokhuehi |
| ◯ | 103 | 3:30PM | Trey Crouch |
| ◯ | 107 | 3:30PM | Marissa Munoz |
| ◯ | 906 | 3:30PM | Andy Yu |
| ◯ | 222 | 4:30PM | Ayo Aitokhuehi |
| ◯ | 101 | 4:30PM | Trey Crouch |
| ◯ | 105 | 4:30PM | Marissa Munoz |
| ◯ | 908 | 4:30PM | Andy Yu |

- Detach the final page of formula sheet from the exam, then write your name and PUID on this cover page, the back cover, and the formula sheet. **DO NOT WRITE ANYTHING ELSE** on these pages. Do not detach any pages other than the formula sheet from the exam booklet.
- This exam consists of 24 questions for a total of 200 points.
- You have exactly two hours to complete the exam.
- Do not open the exam booklet or start writing before the proctor signals the start of the exam.
- Additional pages for scratch work can be found at the end of the booklet.
- Calculators, electronic devices, books, or notes are **NOT ALLOWED**.
- Students may not look at anybody else's exam, and may not communicate with anybody else except with their TA or instructor if there is a question.
- If you finish the exam before 8:55 pm, you may leave the room after turning in the exam booklet. You may not leave the room before 7:20 pm. If you don't finish before 8:55 pm, **YOU MUST REMAIN SEATED** until your TA comes and collects your exam booklet. You must stop working when the proctor signals the end of exam.

*Good luck!*

**DO NOT DETACH THIS PAGE FROM THE EXAM BOOKLET.**

**Answer Sheet Instructions:**

- Fill the circles completely with a **#2 PENCIL** for your answer choices. If you need to change your answer choice, erase the mark completely.

**DO:** ○ ●
**DON'T:** ⊘ ⊗ ⊘ ⊘ ◉ ●

- Mark **EXACTLY ONE** circle for each question. Questions with more than one marked circle will receive no credit.
- Only what you marked on this page will be graded for score.
- Partial credit will not be awarded, unless otherwise indicated by individual questions.

**Mark all your answer choices below:**

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| #1 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |
| #2 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |
| #3 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |
| #4 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |
| #5 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |
| #6 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |
| #7 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |
| #8 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |
| #9 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |
| #10 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |
| #11 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |
| #12 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| #13 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |
| #14 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |
| #15 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |
| #16 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |
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| #20 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |
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| #23 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |
| #24 | Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ | Ⓕ | Ⓖ | Ⓗ | Ⓘ | Ⓙ |

Exam booklet version: ● 01 ○ 02 ○ Alternate ○ Enlarged Text

Make no stray marks in the boxed region above. Do not modify the exam booklet version marking.

**DO NOT DETACH THIS PAGE FROM THE EXAM BOOKLET.**

**Remember to mark all your answer choices on the second cover page!**

1. (8 points) Find the area of the region enclosed by the polar curve

$$r = 2 + \cos(2\theta), \text{ where } 0 \leq \theta \leq 2\pi.$$

(A) $6\pi$
(B) $9\pi/2$
(C) $5\pi$
(D) $9\pi$
(E) $5\pi/2$

2. (8 points) A function $f(x)$ is defined by

$$f(x) = \int_0^x \frac{\sin(t)}{t}\, dt.$$

Which of the following is a power series representation of $f(x)$?

(A) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k)!}$
(B) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+2}}{(2k+2)!}$
(C) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k+1)!}$
(D) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$
(E) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)(2k+1)!}$

**Remember to mark all your answer choices on the second cover page!**

3. (9 points) The following are three attempts at evaluating the integral

$$\int \sin(x)\cos(x)\,dx$$

ATTEMPT (i):

Set

$$u = \sin(x), \quad dv = \cos(x)\,dx; \quad \text{thus} \quad du = \cos(x)\,dx, \quad v = \sin(x).$$

Then we have

$$\int \sin(x)\cos(x)\,dx = uv - \int v\,du = \sin(x)^2 - \int \sin(x)\cos(x)\,dx.$$

From here, we can algebraically solve for the integral, and we obtain

$$\int \sin(x)\cos(x)\,dx = \frac{\sin(x)^2}{2} + C.$$

ATTEMPT (ii):

Set

$$u = \cos(x), \quad \text{then} \quad du = -\sin(x)\,dx,$$

and so

$$\int \sin(x)\cos(x)\,dx = -\int u\,du = -\frac{u^2}{2} + C = -\frac{\cos(x)^2}{2} + C.$$

ATTEMPT (iii):

By the double angle formula $\sin(2x) = 2\sin(x)\cos(x)$, we can rewrite the integral

$$\int \sin(x)\cos(x)\,dx = \frac{1}{2}\int \sin(2x)\,dx = \frac{1}{2}\cdot\frac{-\cos(2x)}{2} + C = -\frac{\cos(2x)}{4} + C.$$

Which of these attempts is/are correct?

Note: Partial credit is possible for this question.

(A) None of the above
(B) Only (i)
(C) Only (ii)
(D) Only (iii)
(E) Only (i) and (ii)
(F) Only (i) and (iii)
(G) Only (ii) and (iii)
(H) All of (i), (ii), and (iii)

**Remember to mark all your answer choices on the second cover page!**

4. (8 points) Evaluate

$$\sum_{n=0}^{\infty} \frac{n+1}{3^n}$$

(A) $9/4$

(B) $3/2$

(C) $\ln(3/2)$

(D) $2$

(E) The series is divergent.

5. (8 points) Given vectors

$$\vec{v} = \langle 1, -1, 0 \rangle \quad \text{and} \quad \vec{w} = \langle -2, 1, 2 \rangle.$$

Compute $\mathsf{proj}_{\vec{v}}(\vec{w})$.

(A) $\langle -3, 3, 0 \rangle$

(B) $\left\langle -\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}, 0 \right\rangle$

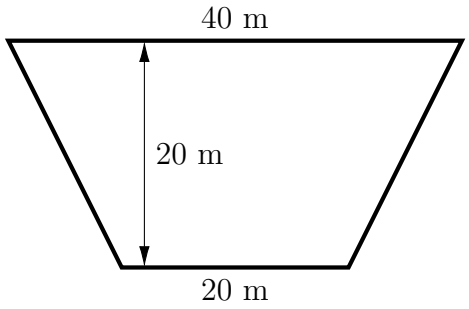
(C) $\left\langle \frac{2}{3}, -\frac{1}{3}, -\frac{2}{3} \right\rangle$

(D) $\left\langle \frac{2}{9}, -\frac{1}{9}, -\frac{2}{9} \right\rangle$

(E) $\left\langle -\frac{3}{2}, \frac{3}{2}, 0 \right\rangle$

**Remember to mark all your answer choices on the second cover page!**

6. (8 points) The face of a dam is in the shape of a trapezoid, which has a 20 m wide base, a 40 m wide top, and a height of 20 m. When the water level is even with the top, which one of the following integrals computes the total hydrostatic force on the face of the dam?
Note: Use $\rho$ kg/m$^3$ for the density of water, and $g$ m/s$^2$ for gravitational acceleration.




(A) $\displaystyle\int_0^{20} \rho g(20+y)y\,dy$

(B) $\displaystyle\int_0^{20} \rho g(40-y)(20+y)\,dy$

(C) $\displaystyle\int_0^{40} \rho g(20+y)y\,dy$

(D) $\displaystyle\int_0^{20} \rho g(40-y)y\,dy$

(E) $\displaystyle\int_0^{40} \rho g(40-y)y\,dy$

7. (8 points) Evaluate

$$\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$$

(A) 1/2

(B) 1

(C) 3/2

(D) 3/4

(E) The series is divergent.

**Remember to mark all your answer choices on the second cover page!**

8. (8 points) Which of the following statements is correct about the polar curve

$$r = \sin(\theta)\cos(\theta)\tan(\theta) \quad ?$$

(A) The curve is symmetric about both the $x$-axis and the $y$-axis, but not about the origin.

(B) The curve is symmetric about the $x$-axis only.

(C) The curve is symmetric about the $x$-axis, the $y$-axis, and the origin.

(D) The curve is symmetric about both the origin and the $y$-axis, but not about the $x$-axis.

(E) The curve is symmetric about the $y$-axis only.

9. (9 points) Given two arbitrary sequences $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$, determine which of the following three statements is/are always TRUE.

(i) If $\{a_n\}_{n=1}^{\infty}$ is convergent, then $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent.

(ii) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are both convergent, then $\{a_n/b_n\}_{n=1}^{\infty}$ is convergent.

(iii) If $\{a_n\}_{n=1}^{\infty}$ is convergent, then $\{|a_n|\}_{n=1}^{\infty}$ is convergent.

Note: Partial credit is possible for this question.

(A) None of the above

(B) Only (i)

(C) Only (ii)

(D) Only (iii)

(E) Only (i) and (ii)

(F) Only (i) and (iii)

(G) Only (ii) and (iii)

(H) All of (i), (ii), and (iii)

**Remember to mark all your answer choices on the second cover page!**

10. (8 points) The power series

$$\sum_{k=0}^{\infty} \frac{(x-1)^k}{2^k(k+1)}$$

has radius of convergence $R = 2$. What is its interval of convergence?

(A) $(-2, 2]$
(B) $(-2, 2)$
(C) $(-1, 3)$
(D) $[-1, 3]$
(E) $[-1, 3)$

11. (8 points) Evaluate

$$\int_0^{\infty} \frac{1}{x^2 + 6x + 9}\, dx$$

(A) $-1/3$
(B) $\pi/3$
(C) $\pi/6$
(D) $1/3$
(E) The integral is divergent.

**Remember to mark all your answer choices on the second cover page!**

12. (8 points) What is the Maclaurin series of the function

$$f(x) = \frac{x^3 \sin(2x)}{2} \quad ?$$

(A) $\displaystyle\sum_{k=0}^{\infty} \frac{(-4)^k\, x^{2k+3}}{(2k)!}$

(B) $\displaystyle\sum_{k=0}^{\infty} \frac{(-4)^k\, x^{2k+4}}{(2k+1)!}$

(C) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k\, x^{2k+4}}{(2k+1)!}$

(D) $\displaystyle\sum_{k=0}^{\infty} \frac{-4^k\, x^{2k+4}}{(2k+1)!}$

(E) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k\, x^{2k+3}}{(2k)!}$

13. (8 points) Evaluate

$$\int_0^{\pi/4} \frac{\sec(x)^2}{\sqrt{\tan(x)}}\, dx$$

(A) $\sqrt{\pi}$

(B) 1

(C) 0

(D) 2

(E) The integral is divergent.

**Remember to mark all your answer choices on the second cover page!**

14. (12 points) Determine which of the following three series is/are convergent.

(i) $\displaystyle\sum_{n=2}^{\infty} \frac{\sin(n)}{n^2+n}$

(ii) $\displaystyle\sum_{n=3}^{\infty} \sqrt{\frac{n^3}{n^5-n}}$

(iii) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\arctan(n)}$

Note: Partial credit is possible for this question.

(A) None of the above
(B) Only (i)
(C) Only (ii)
(D) Only (iii)
(E) Only (i) and (ii)
(F) Only (i) and (iii)
(G) Only (ii) and (iii)
(H) All of (i), (ii), and (iii)

**Remember to mark all your answer choices on the second cover page!**

15. (9 points) Which of the following pairs of polar coordinates and Cartesian coordinates describe the same point in the plane?

    (i) Polar coordinates $(\sqrt{2}, 5\pi/4)$; Cartesian coordinates $(-1, -1)$.

    (ii) Polar coordinates $(-5, 3\pi)$; Cartesian coordinates $(5, 0)$.

    (iii) Polar coordinates $(4, -\pi/3)$; Cartesian coordinates $(2, -2\sqrt{3})$.

    Note: Partial credit is possible for this question.

    (A) None of the above
    (B) Only (i)
    (C) Only (ii)
    (D) Only (iii)
    (E) Only (i) and (ii)
    (F) Only (i) and (iii)
    (G) Only (ii) and (iii)
    (H) All of (i), (ii), and (iii)

16. (8 points) Given that

$$\frac{x-2}{x^3-4x+1} = \frac{A}{x-r_1} + \frac{B}{x-r_2} + \frac{C}{x-r_3},$$

    where $r_1$, $r_2$, and $r_3$ are three distinct real roots of the polynomial $x^3 - 4x + 1$. What is the value of $A + B + C$?

    Hint: Do not try to find the values of $r_1$, $r_2$, and $r_3$.

    (A) 0
    (B) 1
    (C) 2
    (D) $-2$
    (E) $-1$

**Remember to mark all your answer choices on the second cover page!**

17. (9 points) Which of the following polar equations describe a straight line?

(i) $r = \sin(\theta)$.

(ii) $r = \cos(\theta) - \sin(\theta)$.

(iii) $r = \sec(\theta)$.

Note: Partial credit is possible for this question.

(A) None of the above
(B) Only (i)
(C) Only (ii)
(D) Only (iii)
(E) Only (i) and (ii)
(F) Only (i) and (iii)
(G) Only (ii) and (iii)
(H) All of (i), (ii), and (iii)

18. (8 points) Consider the following four points

$$A = (-2, 8), \quad B = (1, 4), \quad C = (3, -4), \quad \text{and} \quad D = (-9, 5).$$

Suppose that we have two circle:

- The first circle is centered at point $A$, with point $B$ on the first circle;
- The second circle is centered at point $C$, with point $D$ on the second circle.

Which one of the following statements about these two circles is true?

(A) The circles intersect at two points.
(B) The circles are disjoint, and one is enclosed inside of the other.
(C) The circles are tangent, and they are outside of each other.
(D) The circles are tangent, and one is enclosed inside of the other.
(E) The circles are disjoint, and they are outside of each other.

**Remember to mark all your answer choices on the second cover page!**

19. (8 points) What is the radius of convergence $R$ of the power series

$$\sum_{k=1}^{\infty} \frac{k!\, x^{k+4}}{(3k)^k} \quad ?$$

(A) $R = 3e$
(B) $R = \frac{1}{3e}$
(C) $R = \frac{1}{3}$
(D) $R = 3$
(E) $R = \frac{3}{e}$

20. (8 points) Find the arc length of the polar curve

$$r = e^{\theta}, \text{ where } 0 \leq \theta \leq 1.$$

(A) $e - 1$
(B) $\sqrt{2}(e - 1)$
(C) $\sqrt{2}e$
(D) $2(e - 1)$
(E) $e^2 - 1$

**Remember to mark all your answer choices on the second cover page!**

21. (8 points) Which of the following conclusion is correct if the root test is applied to the series

$$\sum_{n=3}^{\infty} \left(\frac{2}{n} - 1\right)^{n^2} \quad ?$$

(A) The series is divergent because $\rho = e^2$.

(B) The series is absolutely convergent because $\rho = 0$.

(C) The series is divergent because $\rho = \infty$.

(D) The series is absolutely convergent because $\rho = e^{-2}$.

(E) The root test is inconclusive because $\rho = 1$.

22. (8 points) A function $f(x)$ is defined by the power series

$$f(x) = \sum_{k=0}^{\infty} \frac{x^{2k}}{k^3 - 4}.$$

What is its fourth-derivative $f^{(4)}(0)$?

(A) $1/60$

(B) $0$

(C) $1/4$

(D) $2/5$

(E) $6$

**Remember to mark all your answer choices on the second cover page!**

23. (8 points) A function $f(x)$ is defined by the power series

$$f(x) = \sum_{k=1}^{\infty} \frac{x^{2k}}{(2k)!}.$$

Evaluate

$$\lim_{x \to 0} \frac{f(2x)}{f(x)}.$$

(A) 2

(B) 0

(C) 4

(D) 1

(E) The limit does not exist.

24. (8 points) A hemispherical dome has height $h = 2$ m. The top half of the dome (in height) needs a paint job. What is the total area that is being painted?

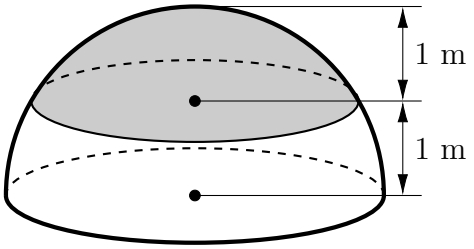
(A) $4\pi$ m$^2$

(B) $4\sqrt{3}\pi$ m$^2$

(C) $2\sqrt{3}\pi$ m$^2$

(D) $2\pi$ m$^2$

(E) $\sqrt{3}\pi$ m$^2$



**Remember to mark all your answer choices on the second cover page!**
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**Remember to mark all your answer choices on the second cover page!**
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Write down your name and PUID on the back.

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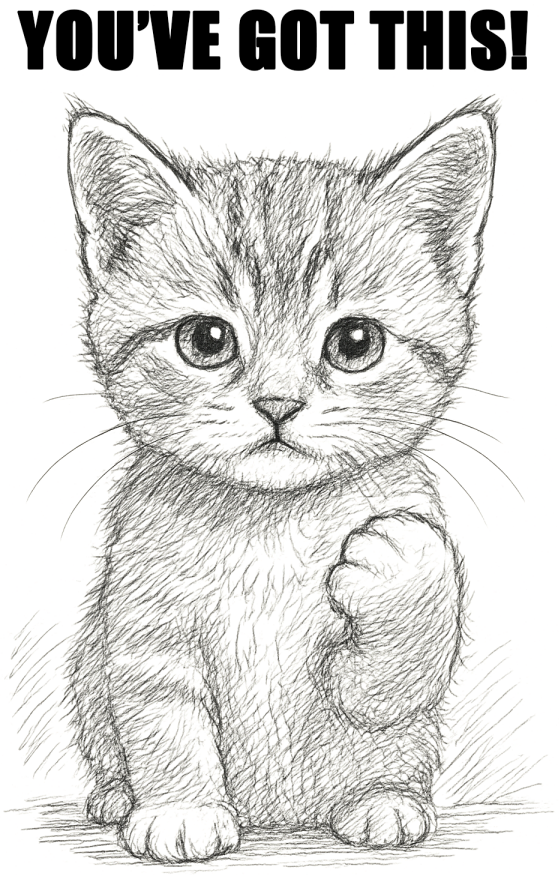


Do not write anything on this page.

**Remember to mark all your answer choices on the second cover page!**
**DETACH THIS FORMULA SHEET FROM THE EXAM BOOKLET.**
Write down your name and PUID on this page.

Name: _____ PUID #: _____

**Common Maclaurin series:**

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \quad \text{for } -\infty < x < \infty$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \quad \text{for } -\infty < x < \infty$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}, \quad \text{for } -\infty < x < \infty$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k}, \quad \text{for } -1 < x \leq 1$$

$$\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}, \quad \text{for } -1 \leq x \leq 1$$

**Common trigonometric identities:**

$$\cos(x)^2 + \sin(x)^2 = 1$$

$$\sec(x)^2 - \tan(x)^2 = 1$$

$$\sin(2x) = 2\cos(x)\sin(x)$$

$$\cos(2x) = \cos(x)^2 - \sin(x)^2$$

$$\cos(2x) = 2\cos(x)^2 - 1$$

$$\cos(2x) = 1 - 2\sin(x)^2$$