MATH 162 – SPRING 2004 – FINAL EXAM
MAY 3, 2004

STUDENT NAME————————————————

STUDENT ID————————————————

RECITATION INSTRUCTOR————————————————

RECITATION TIME————————————————

INSTRUCTIONS

1. Fill in your name, your student ID number, and your recitation instructor's name and recitation time above. Write your name, your student ID number and division and section number of your recitation section on your answer sheet, and fill in the corresponding circles.
2. Mark the letter of your response for each question on the mark-sense sheet.
3. There are 25 questions, each worth 8 points.
8. No books, notes or calculators may be used.

Useful formulas:

Trigonometry:

$$1 + \tan^2 x = \sec^2 x$$

$$\cos(\frac{\pi}{3}) = \sin(\frac{\pi}{6}) = \frac{1}{2}, \quad \cos(\frac{\pi}{6}) = \sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}, \quad \cos(\frac{\pi}{4}) = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$$

Moments and center of mass:

$$M_x = \int_a^b \frac{1}{2}\left((f(x))^2 - (g(x))^2\right) dx, \quad M_y = \int_a^b x\,(f(x) - g(x))\,dx$$

$$\overline{x} = \frac{M_y}{M}, \quad \overline{y} = \frac{M_x}{M},$$

Arc length

$$L = \int_a^b \sqrt{1 + (f'(x))^2} \ dx \quad \text{or}$$

$$L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} \, dt.$$

Area of a surface of revolution

$$S = \int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2} \, dx \quad \text{or}$$

$$\text{Area} \ = \int_a^b 2\pi y(t) \sqrt{(x'(t))^2 + (y'(t))^2} \, dt$$

Area in polar coordinates

$$\text{Area} = \frac{1}{2} \int_\alpha^\beta (r(\theta))^2 \, d\theta$$

Length in polar coordinates

$$L = \int_\alpha^\beta \sqrt{r(\theta)^2 + (r'(\theta))^2} \, d\theta,$$

Other formulas:

$$\text{Work} = \int_a^b F(x)dx, \quad \text{Hooke's law: } F(x) = kx.$$

Taylor series

$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2!} f^{(2)}(a)(x-a)^2 + \frac{1}{3!} f^{(3)}(a)(x-a)^3 + \ldots + \frac{1}{n!} f^{(n)}(a)(x-a)^n + r_n(x)$$

$$\text{where} \quad r_n(x) = \frac{1}{(n+1)!} f^{(n+1)}(t_x)(x-a)^{n+1} \quad \text{and } t_x \text{ is between } a \text{ and } x$$

$$\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1},$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!},$$

1) The area of the parallelogram which is spanned by the vectors $\vec{a} = \vec{i} - 2\vec{j} + 2\vec{k}$ and $\vec{b} = 3\vec{j} - \vec{k}$ is

A) 5

B) $2\sqrt{2}$

C) 4

D) $\sqrt{26}$

E) 6

2) The angle between the vectors $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} + \mathbf{k}$ is

A) $\frac{\pi}{4}$

B) $\frac{\pi}{2}$

C) $\frac{\pi}{3}$

D) π

E) 0

3) Compute

$$\int_0^1 xe^x \, dx$$

A) 1

B) 0

C) e

D) -1

E) $e - 1$

4) Compute the improper integral

$$\int_e^\infty \frac{1}{x(\ln x)^2} \, dx$$

A) $\frac{1}{e}$

B) It diverges

C) 1

D) e^2

E) $\frac{1}{e^2}$

5)

$$\int_0^{\frac{\pi}{4}} (\sec x)^2 (\tan x)^2 \; dx =$$

A) $\frac{1}{3}$

B) $\frac{1}{2}$

C) $\frac{2}{3}$

D) $\frac{1}{4}$

E) 1

6) The substitution best suited for computing $\int \sqrt{x^2 + 2x + 5} \;\; dx$ is

A) $x = 2 \sin u$

B) $x = -1 = 2 \sec u$

C) $x = \sec(u + 1)$

D) $x = -1 + 2 \tan u$

E) $x = 2 \tan(u + 1)$

7) Compute $\int_3^4 \frac{3x-4}{x^2-3x+2}\ dx$

A) $\ln 3$

B) $\ln 12$

C) $\ln 6$

D) $\ln \frac{3}{4}$

E) $\ln \frac{2}{3}$

8) Let R be the region between $y = \frac{1}{x}$, $y = 0$, $x = 1$ and $x = 3$. The volume of the solid generated by revolving R about the line $y = -1$ is given by

A) $\pi \int_1^3 \left(\frac{1}{x} + 1\right)^2\ dx$

B) $\pi \int_1^3 \left(\frac{1}{x^2} + 1\right)\ dx$

C) $\pi \int_1^3 \left(\frac{1}{x^2} - 1\right)\ dx$

D) $\pi \int_1^3 \left(\left(\frac{1}{x} + 1\right)^2 - 1\right)\ dx$

E) $\pi \int_1^3 \left(\left(\frac{1}{x} + 1\right)^2 + 1\right)\ dx$

9) Which of the following integrals represnt the length of the arc of the parabola $y = x^2$, $0 \leq x \leq \frac{1}{2}$.

A) $\int_0^{\frac{\pi}{4}} \sin^2 u \, du$

B) $\frac{1}{2} \int_0^{\frac{\pi}{4}} \sec^3 u \, du$

C) $2 \int_0^{\frac{\pi}{4}} \sec^3 u \, du$

D) $\int_0^{\frac{\pi}{3}} \tan^2 u \, du$

E) $2 \int_0^{\frac{\pi}{3}} \sec^3 u \, du$

10) A tank of height 10 feet and whose horizontal cross sections are squares of side 2 feet is filled with water. How much work is required to pump out all but 2 feet of water to the top of the tank? Water weighs $62.5\,\mathrm{lb} / \mathrm{ft}^3$

A) 9600 ftlb

B) 6400 ftlb

C) 10000 ftlb

D) 8000 ftlb

E) 7200 ftlb

11) It takes a force of 4 lbs to hold a spring 6 inches beyond its natural length. Find the amount of work is required to stretch it from 6 inches to 1 foot from its neutral position.

A) 2 ft-lb

B) 3 ft-lb

C) 4 ft-lb

D) 6 ft-lb

E) 8 ft-lb

12) The center of gravity of the region in the plane bounded by $y = 2x$, $y = 0$ and $x = 1$ is

A) $(\frac{4}{3}, \frac{1}{3})$

B) $(\frac{1}{3}, \frac{2}{3})$

C) $(\frac{2}{3}, \frac{2}{3})$

D) $(\frac{2}{3}, \frac{1}{3})$

E) $(0, 0)$

13)

$$\sum_{n=2}^{\infty}\left(-\frac{2}{3}\right)^n =$$

A) $\frac{3}{5}$

B) $-\frac{3}{5}$

C) $-\frac{2}{5}$

D) $\frac{4}{15}$

E) $-\frac{4}{15}$

14) The series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

A) diverges by the ratio test

B) converges by the ratio test

C) converges because $\lim_{n\to\infty} \sin(\frac{1}{n}) = 0$

D) converges by the limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

E) diverges by the limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

15) Given two series

$$S_1 = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+2}}, \quad S_2 = \sum_{n=10000}^{\infty} \frac{1}{n\ln(n)}$$

A) Only S_2 converges

B) Only S_1 converges

C) Both converge

D) Neither converge

E) $S_1 + S_2$ converges

16) The limit

$$\lim_{n\to\infty} \frac{\ln(2+e^n)}{2n} =$$

A) 1

B) $\frac{1}{2}$

C) 2

D) e

E) e^2

17) The radius of convergence of

$$\sum_{n=1}^{\infty} \frac{e^n x^n}{n!}$$

A) is $\frac{1}{e}$

B) is e

C) is 0

D) is ∞

E) is 1

18)

$$\int_0^x e^{t^2}\, dt =$$

A) $\sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)!}$

B) $\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

C) $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!(2n+1)}$

D) $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!}$

E) $e^{x^2} - 1$

19) The first three terms of the MacLaurin series of $f(x) = x\ln(1+x^2)$ are

A) $x - \frac{x^3}{3} + \frac{x^5}{5}$

B) $x^2 - \frac{x^4}{2} + \frac{x^6}{3}$

C) $x^2 - \frac{x^4}{2} + \frac{x^6}{6}$

D) $x^3 - \frac{x^5}{2} + \frac{x^7}{3}$

E) $x^3 - \frac{x^5}{5} + \frac{x^7}{7}$

20) The area of the region inside $r = 3\sin\theta$ and outside $r = 2 - \sin\theta$ is given by

A) $\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(4\sin^2\theta + 2\sin\theta - 2\right) \; d\theta$

B) $\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \left(8\sin^2\theta + 4\sin\theta - 4\right) \; d\theta$

C) $\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(8\sin^2\theta + 4\sin\theta - 4\right) \; d\theta$

D) $\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \left(4\sin^2\theta + 2\sin\theta - 2\right) \; d\theta$

E) $\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(4\sin^2\theta + 2\sin\theta + 2\right) \; d\theta$

21) Find the slope of the tangent to the astroid $x = 2\cos^3\theta$, $y = 2\sin^3\theta$ at $\theta = \frac{\pi}{3}$.

A) $-\frac{\sqrt{2}}{2}$

B) $\frac{1}{\sqrt{3}}$

C) $-\frac{1}{\sqrt{3}}$

D) $\sqrt{3}$

E) $-\sqrt{3}$

22) The length of the curve with polar equation $r = \cos\theta - \sin\theta$, $0 \leq \theta \leq \frac{\pi}{4}$ is

A) $\frac{1}{2}$

B) $\frac{\pi}{2}$

C) $\frac{\pi\sqrt{2}}{4}$

D) $\frac{\sqrt{2}}{2}$

E) $\frac{3\pi\sqrt{2}}{2}$

23) Find an equation for the conic section whose foci are $(5, 0)$ and $(-5, 0)$ and eccentricity is $\frac{5}{3}$

A) $\frac{x^2}{25} - \frac{y^2}{16} = 1$

B) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

C) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

D) $\frac{x^2}{16} + \frac{y^2}{25} = 1$

E) $\frac{x^2}{9} + \frac{y^2}{16} = 1$

24) Find an equation for the directrix of the parabola $x^2 + 12x - 2y + 40 = 0$

A) $y = 2$

B) $y = \frac{5}{2}$

C) $y = \frac{3}{2}$

D) $x = -6$

E) $x = -\frac{9}{2}$

25) A conic section has the polar equation

$$r = \frac{3}{2 + 4\cos\theta}.$$

The type of this conic section and its directrix are:

A) A parabola with directrix $x = 3$

B) An ellipse with directrix $x = \frac{3}{4}$

C) A hyperbola with directrix $x = 3$

D) A hyperbola with directrix $x = \frac{3}{4}$

E) An ellipse with directrix $x = 3$