MA 16200

FINAL EXAM INSTRUCTIONS

VERSION 01

Name ______________________________

10–digit PUID number ______________________________

Recitation Instructor ______________________________

Recitation Section Number and Time ______________________________

Lecturer ______________________________

Instructions:

1. Do not open this booklet until you are instructed to.
2. Fill in all the information requested above and on the scantron sheet. On the scantron sheet fill in the little circles for your name, section number and PUID.
3. This booklet contains 25 problems. The maximum score is 200 points.
4. For each problem mark your answer on the scantron sheet and also **circle it in this booklet**.
5. Work only on the pages of this booklet.
6. Books, notes, calculators or any electronic device are not allowed during this test and they should not even be in sight in the exam room. You may not look at anybody else's test, and you may not communicate with anybody else, except, if you have a question, with your instructor.
7. You are not allowed to leave during the first 20 and the last 5 minutes of the exam.
8. When time is called at the end of the exam, put down your writing instruments and remain seated. The TAs will collect the scantrons and the booklets.

1. The area of the triangle with vertices $(0,0,0)$, $(2,-1,2)$ and $(4,1,3)$ is

A) $\dfrac{\sqrt{41}}{2}$

B) $\dfrac{\sqrt{65}}{2}$

C) $2\sqrt{65}$

D) $\dfrac{\sqrt{61}}{2}$

E) None of the above.

2. Evaluate $\displaystyle\int_1^e x^2 \ln x dx$.

A) $\dfrac{e^3+1}{3}$

B) $\dfrac{e^3}{3}$

C) $\dfrac{e^2+e}{9}$

D) $\dfrac{2e^3}{9}$

E) $\dfrac{2e^3+1}{9}$

3. If $\vec{u} = b\vec{i} + 2\vec{j} - \vec{k}$ and $\vec{v} = \vec{i} + a\vec{j} - 2a\vec{k}$ are perpendicular, then we must have

A) $a = -b$

B) $a = 4b$

C) $a = \dfrac{-b}{4}$

D) $b = -3a$

E) $b = \dfrac{-a}{3}$

4. The region bounded by $y = 2x$, $x = 2$ and the x-axis is rotated about the line $x = 3$. The volume of the resulting solid is given by (use the cylindrical shell method).

A) $\displaystyle\int_0^2 2\pi(3-x)(2x-3)dx$

B) $\displaystyle\int_0^2 2\pi(3-2x)(x)dx$

C) $\displaystyle\int_0^2 2\pi(3-2x)dx$

D) $\displaystyle\int_0^2 2\pi(3-x)(2x-2)dx$

E) $\displaystyle\int_0^2 2\pi(3-x)(2x)dx$

5. If the region bounded by $y = \sin x$, $y = 0$ and $0 \leq x \leq \pi$ is rotated around the x-axis, then find the volume of the resulting solid.

A) $\dfrac{\pi^2}{2}$

B) $\dfrac{2\pi}{3}$

C) π^2

D) $\dfrac{4\pi}{3}$

E) $\dfrac{3\pi^2}{4}$

6. The work required to stretch a spring 4 ft beyond its natural length is 24 ft-lb. How much is needed to stretch it 2 ft beyond its natural length?

A. 8 ft-lb.

B. $\dfrac{9}{2}$ ft-lb.

C. $\dfrac{16}{3}$ ft-lb.

D. $\dfrac{32}{31}$ ft-lb.

E. None of the above

7. The length of the curve $y = 3 + \frac{2}{3}\, x^{3/2}$ from $x = 0$ to $x = 1$ is

A. $\frac{2}{3}(5^{3/2} - 1)$

B. $\frac{4}{9}(13^{3/2} - 8)$

C. $2(2^{3/2} - 1)$

D. $\frac{2}{3}(2^{3/2} - 1)$

E. $2(5^{3/2} - 1)$

8. The partial fraction decomposition of $\dfrac{3x+2}{(x^2+2)(x^4-16)}$ is

A. $\dfrac{A}{x^2+2} + \dfrac{Bx+C}{x^4-16}$

B. $\dfrac{Ax+B}{x^2+2} + \dfrac{Cx+D}{x^2+4} + \dfrac{F}{x-2}$

C. $\dfrac{A}{(x-2)} + \dfrac{B}{x+2} + \dfrac{Cx+D}{x^2+4} + \dfrac{Ex+F}{x^2+2}$

D. $\dfrac{A}{(x-2)} + \dfrac{B}{x+2} + \dfrac{Cx+D}{x^2+2}$

E. None of the above.

9. An appropriate trig substitution will convert the definite integral $\int_1^3 \sqrt{x^2+2x-3}\,dx$ into which of the following definite integrals?

A. $\int_0^{\pi/3} 4\sec\theta\tan\theta d\theta$

B. $\int_0^{\pi/3} 4\sec\theta\tan^2\theta d\theta$

C. $\int_0^{\pi/3} 2\sec\theta\tan^3\theta d\theta$

D. $\int_0^{\pi/6} 2\sec\theta\tan^2\theta d\theta$

E. $\int_0^{\pi/3} \sec\theta\tan\theta d\theta$

10. $\int \frac{x}{\sqrt{x^2+2x+2}}\,dx =$

A. $\sqrt{x^2+2x+2} - \ln\left|\sqrt{x^2+2x+2}+x+1\right| + C$

B. $\sqrt{x^2+2x+2} + \ln|x^2+2x+1| + C$

C. $\ln\left|\sqrt{x^2+2x+2}+x+1\right| + C$

D. $\frac{1}{2}\sqrt{x^2+2x+2} - \frac{1}{2}\ln\left|\sqrt{x^2+2x+2}+x+1\right| + C$

E. None of the above.

11. The area between the graphs $x = 3 - y^2 + 2y$ and $x = y^2 - 2y + 3$ is

A. $\frac{8}{5}$

B. $\frac{8}{3}$

C. $\frac{7}{3}$

D. $\frac{13}{3}$

E. $\frac{14}{5}$

12. Which of the following sequences will converge?

(i) $\left\{\frac{n+1}{n-1}\right\}$ (ii) $\left\{(-1)^n + \frac{1}{n}\right\}$ (iii) $\{n(n-1)\}$ (iv) $\left\{\frac{n!}{(n+1)!}\right\}$ (v) $\left\{\frac{n!}{2^n}\right\}$

A. all diverge

B. only (ii) and (v)

C. (i), (iv) and (v)

D. only (i) and (iv)

E. (iv) and (v)

13. Compute $\sum_{n=1}^{\infty} \frac{(-2)^{n+1}}{3^n}$

A. 4

B. $\frac{12}{5}$

C. $-\frac{4}{5}$

D. $\frac{4}{5}$

E. -4

14. Compute $\frac{2+3i}{2-i}$.

A. $\frac{1+8i}{5}$

B. $\frac{1-8i}{5}$

C. $\frac{7-8i}{5}$

D. $\frac{7+8i}{5}$

E. $\frac{1-8i}{25}$

15. Given the series I. $\sum_{n=1}^{\infty} \frac{1+\ln n}{n}$ and II. $\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{n^2}$

which one of the following statements using the Comparison Test is true?

A. I converges by comparing it to $\sum_{n=1}^{\infty} \frac{1}{n}$

B. I diverges by comparing it to $\sum_{n=1}^{\infty} \frac{1}{n}$

C. II converges by comparing it to $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

D. II diverges by comparing it to $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

E. II diverges by comparing it to $\sum_{n=1}^{\infty} \frac{1}{n^2}$

16. Which of the following series is conditionally convergent?

A. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

B. $\sum_{n=1}^{\infty} \left(1 + \frac{(-1)^n}{n}\right)$

C. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+n}}$

D. $\sum_{n=1}^{\infty} (-1)^n \frac{e^n}{\sqrt{n}}$

E. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

17. The set of all x for which the power series $\sum_{n=1}^{\infty} \frac{2^n}{\sqrt{n}} x^n$ converges is

A. $-\frac{1}{2} \leq x < \frac{1}{2}$

B. $-\frac{1}{2} < x < \frac{1}{2}$

C. $-\frac{1}{2} \leq x \leq \frac{1}{2}$

D. $x = 0$ only

E. $-\infty < x < \infty$

18. The Maclaurin series for the function $f(x) = \frac{x}{(1+x^2)^2}$ is

A. $\sum_{n=1}^{\infty} (-1)^n x^{2n}$

B. $\sum_{n=1}^{\infty} (-1)^n 2n x^{2n-1}$

C. $\sum_{n=1}^{\infty} (-1)^n n x^{2n-1}$

D. $\sum_{n=1}^{\infty} (-1)^{n-1} n x^{2n-1}$

E. $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$

19. If $f(x) = xe^x$, evaluate $f^{(6)}(0)$

A. $6!$

B. $5!$

C. 6

D. 5

E. $\dfrac{7!}{6}$

20. Compute $\displaystyle\int \frac{e^{2x}-1}{x}\,dx$

A. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n x^n}{n!n} + C$

B. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n x^{n+1}}{(n+1)!} + C$

C. $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n+1} x^n}{n!n} + C$

D. $\displaystyle\sum \frac{2^n x^n}{n!} + C$

E. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n x^n}{n!} + C$

21. If $(1+x^2)^{4/3} = \sum_{n=1}^{\infty} c_n x^n$, compute c_6

A. $-\frac{4}{27}$

B. $\frac{4}{81}$

C. $-\frac{8}{27}$

D. $\frac{4}{27}$

E. $-\frac{4}{81}$

22. Let $x = t^2$, $y = t^2 + t$. Find $\frac{d^2y}{dx^2}$ at the point $(1, 2)$.

A. $1/2$

B. $-1/2$

C. $-1/4$

D. $1/4$

E. 2

23. Which equation corresponds to the curve sketched below.

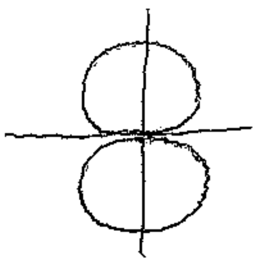
A. $r^2 = \cos^2\theta$

B. $r = \sin\theta$

C. $r = \sin^2\theta$

D. $r = \sin 2\theta$

E. $r = \cos^2\theta$



24. Find the foci of the ellipse
$4x^2 + y^2 - 2y = 15$.

A. $(1, \pm\sqrt{20})$

B. $(0, 1 \pm \sqrt{20})$

C. $(0, 1 \pm \sqrt{12})$

D. $(\pm\sqrt{12}, 1)$

E. $(0, -3)$ and $(0, 5)$

25. Which of the following statements is/are true?

(i) By using the Limit Comparison Test with $\sum b_n = \sum \frac{1}{n}$, it follows that $\sum_{n=1}^{\infty} \frac{n}{n^2+4}$ diverges.

(ii) The Integral Test implies that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges.

(iii) The Root Test implies that $\sum_{n=2}^{\infty} (1 - \frac{1}{n})^n$ converges.

A. (i) and (iii)

B. Just (i)

C. Just (iii)

D. Just (ii) and (iii)

E. None are true