MA162 — FINAL EXAM — SPRING 2017 — MAY 03, 2017
TEST NUMBER 01

INSTRUCTIONS:

1. **Do not open the exam booklet until you are instructed to do so.**
2. **Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
3. **MARK YOUR TEST NUMBER ON YOUR SCANTRON**
4. Once you are allowed to open the exam, make sure you have a complete test. There are 15 different test pages (including this cover page).
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. **Circle your answers on this test booklet.**
6. Each problem is worth 8 points. The maximum possible score is 200 points. No partial credit.
7. Do not leave the exam room during the first 20 minutes of the exam.
8. If you do not finish your exam in the first 110 minutes, you must wait until the end of the exam period to leave the room.
9. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

DON'T BE A CHEATER:

1. Do not give, seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructor.
2. Do not look at the exam or scantron of another student.
3. Do not allow other students to look at your exam or your scantron.
4. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
5. Do not consult notes or books.
6. **Do not handle** phones or cameras, calculators or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

STUDENT ID NUMBER: ______________________________

SECTION NUMBER AND RECITATION INSTRUCTOR: ______________________________

USEFUL FORMULAS

Trig Formulas:

$$\sin^2 x = \frac{1-\cos(2x)}{2}, \qquad \cos^2 x = \frac{1+\cos(2x)}{2}, \qquad \sec^2 x = 1 + \tan^2 x$$

Center of Mass:

$$\overline{x} = \frac{1}{A}\int_a^b x(f(x)-g(x))\,dx \quad \text{and} \quad \overline{y} = \frac{1}{A}\int_a^b \frac{1}{2}[(f(x))^2-(g(x))^2]\,dx$$

Arc Length, Surface Area and Volume:

Arc Length: $L = \int_a^b \sqrt{1+(f'(x))^2}\,dx$

Surface area: $S = 2\pi\int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx$

Volume by the washer method: $V = \pi\int_a^b (R^2(x) - r^2(x))\,dx$; $R(x)$ and $r(x)$ are the longer and shorter radii of the washer

Volume by cylindrical shells: $V = 2\pi\int_a^b xf(x)\,dx$

Maclaurin Series:

The geometric series: $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, provided $|x| < 1$

The exponential function: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for all x

Sine: $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ for all x

Cosine: $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ for all x

The binomial series: Provided $|x| < 1$,

$$(1+x)^k = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \ldots + \frac{k(k-1)(k-2)\ldots(k-n+1)}{n!}x^n \ldots$$

1. Find the area of the triangle with vertices $A(1,1,1),\ B(3,2,2),\ C(2,3,1)$.

A. 2

B. $\dfrac{\sqrt{7}}{2}$

C. $\sqrt{14}$

D. $\dfrac{\sqrt{14}}{2}$

E. $\sqrt{7}$

2. Find the values of x such that the vectors $x\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $x\mathbf{i}+4\mathbf{j}+2x\mathbf{k}$ are orthogonal.

A. $x=-2$ and $x=-4$

B. $x=-4$ and $x=-8$

C. $x=2$ and $x=-4$

D. $x=0$ and $x=3/2$

E. $x=0$ and $x=3$

3. Find the area of the region bounded by $y = x^2$ and $y = 6x - 2x^2$.

A. $\frac{1}{3}$

B. $\frac{8}{3}$

C. 4

D. 6

E. $\frac{20}{3}$

4. What can be said about the integral $\displaystyle\int_0^1 4x^3 \ln(x)\, \mathrm{d}x$?

A. It converges and it is equal to -1

B. It diverges

C. It converges and it is equal to $-\frac{1}{4}$

D. It converges and it is equal to $-\frac{3}{4}$

E. It converges and it is equal to $-\frac{1}{3}$

5. Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \cos^3\theta \sin\theta \, d\theta$.

A. $\dfrac{1}{4}$

B. $\dfrac{3}{4}$

C. $\dfrac{1}{2}$

D. $\dfrac{2}{3}$

E. $\dfrac{1}{3}$

6. Compute the length of the arc of the curve given by $y(x) = \displaystyle\int_0^x \sqrt{\cos 2t} \, dt$ from $x = 0$ to $x = \dfrac{\pi}{4}$. (The formula for the arc length can be found on page 2. Hint: use one of the trig formulas also given on page 2).

A. $2\sqrt{2}$

B. $\dfrac{\sqrt{2}}{2}$

C. 1

D. $\dfrac{1}{2}$

E. 2

7. The area of the region bounded by $y = \tan x$ and $y = 0$, for $0 \le x \le \frac{\pi}{4}$ is equal to $A = \frac{\ln 2}{2}$. The y-coordinate of the center of mass of this region is equal to (the formulas for the center of mass are given on page 2).

A. $\frac{1}{2A}(\pi - 1)$

B. $\frac{1}{2A}(1 - \frac{\pi}{8})$

C. $\frac{\pi}{2A}$

D. $\frac{1}{2A}(1 - \frac{\pi}{4})$

E. $\frac{1}{2A}(2\pi - 3)$

8. The area bounded by the curves $y = \sin x$ and $y = 0$, for $0 \le x \le \pi$, is rotated about the y-axis. The volume of the resulting solid is equal to

A. π^2

B. $4\pi^2$

C. 2π

D. 4π

E. $2\pi^2$

9. Using the washer method, the volume of the solid generated by revolving the region bounded by $y = x - 1$, $x = 2$ and the x-axis about the y-axis is:

A. $\pi \int_0^2 (x-1)^2 \, dx$

B. $\pi \int_1^2 (x-1)^2 \, dx$

C. $\pi \int_1^2 [(x-1)^2 - 4] \, dx$

D. $\pi \int_0^1 [(y+1)^2 - 4] \, dy$

E. $\pi \int_0^1 [4 - (y+1)^2] \, dy$

10. A force of 10 lb holds a spring 2 ft beyond its natural length. How much work does it take to stretch the spring from its natural length to 2 ft? (The answer is expressed in ft-lb)

A. 5

B. 10

C. 20

D. 5/2

E. 1

11. The integral $\int_0^1 \frac{1}{x^2+3x+2}\,dx$ equals

A. $\ln\frac{5}{4}$

B. $\ln\frac{4}{3}$

C. $\ln 3$

D. $\ln\frac{8}{3}$

E. $\ln 2$

12. Compute the integral $\int_0^1 \frac{2}{(x+1)(x^2+1)}\,dx$

A. $\frac{1}{2}(\pi + \ln 2)$

B. $\frac{1}{2}(\frac{\pi}{2} + \ln 2)$

C. $\frac{1}{2}(\pi + 2\ln 2)$

D. $\frac{1}{2}(\pi - \ln 2)$

E. $\frac{1}{2}(\frac{\pi}{2} - \ln 2)$

13. Find the exact area of the surface obtained by rotating the curve $y = \frac{1}{3}x^3$, $0 \le x \le 1$, about the x-axis (the formula for the surface area is given on page 2 of the exam).

A. $\frac{\pi}{9}(2\sqrt{2}+1)$

B. $\frac{\pi}{9}(\sqrt{2}+1)$

C. $\frac{\pi}{9}(2\sqrt{2}-1)$

D. $\frac{\pi}{9}(\sqrt{2}-1)$

E. $\frac{\pi}{9}(3\sqrt{2}-1)$

14. Compute the limit $\lim_{n\to\infty} \frac{\frac{\pi}{2} - \arctan n}{\frac{1}{n}}$

A. 1

B. -1

C. $\frac{1}{2}$

D. $\frac{3}{2}$

E. $\frac{1}{4}$

15. Determine the radius and interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{2^n(x-3)^n}{\sqrt{n+3}}.$$

A. $R = 1/2$, interval $\left[\frac{5}{2}, \frac{7}{2}\right)$

B. $R = 1/2$, interval $\left(\frac{5}{2}, \frac{7}{2}\right]$

C. $R = 2$, interval $[1, 5)$

D. $R = 2$, interval $(1, 5]$

E. $R = \infty$, interval $(-\infty, \infty)$

16. Suppose the power series $\sum_{n=0}^{\infty} a_n(x-2)^n$ converges for $x = 4$ and diverges for $x = 6$. Which of the following statements are true?

(1) The power series diverges for $x = 7$.

(2) The power series converges for $x = 0.5$.

(3) The power series diverges for $x = 1$.

(4) The power series converges for $x = -3$.

A. All of them

B. None of them

C. (1)

D. (1) and (2)

E. (1), (2), and (3)

17. The Taylor series of $f(x) = \dfrac{1}{x^2}$ at $x = 1$ is

A. $\displaystyle\sum_{n=0}^{\infty} (n+1)!\ (x-1)^n$

B. $\displaystyle\sum_{n=0}^{\infty} (-1)^n (n+1)!\ (x-1)^n$

C. $\displaystyle\sum_{n=0}^{\infty} (-1)^{n+1} (n+1)\ (x-1)^n$

D. $\displaystyle\sum_{n=0}^{\infty} (n+1)\ (x-1)^n$

E. $\displaystyle\sum_{n=0}^{\infty} (-1)^n (n+1)\ (x-1)^n$

18. The alternating series $\displaystyle\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n \ln n}$ is:

A. Absolutely convergent

B. Conditionally convergent

C. Conditionally and absolutely convergent

D. Divergent

E. Cannot be determined

19. What is the value of r such that $4\sum_{n=0}^{\infty} r^n = 3$?

A. $r = 1/2$

B. $r = 2/3$

C. $r = -1/3$

D. $r = 1/3$

E. $r = -2/3$

20. The terms of a series $\sum_{n=1}^{\infty} a_n$ are defined recursively by the equations

$$a_1 = 4, \qquad a_{n+1} = \frac{(3n^2+2)a_n}{n^p}.$$

For what values of p does the series converge?

A. For all $p > 0$

B. For all $p \geq 1$

C. For all $p > 1$

D. For all $p \geq 2$

E. For all $p > 2$

21. What can be said about the following statements?

I) The series $\displaystyle\sum_{n=1}^{\infty} \frac{n^6+8n}{n^8+4n+10}$ converges by limit comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$.

II) The series $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{n^2}$ converges by limit comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$.

III) The series $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{n^2+8}{n^5+2}}$ converges by limit comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$

A. I and III are false, II is true

B. I is false, II and III are true

C. I and II are false, III is true

D. I, II and III are true

E. I, II and III are false

22. If we use the approximation $\cos(0.1) \sim 1 - \dfrac{1}{200}$, and if we use the alternating series estimate, what is the best we can say about the error of this approximation?

A. The error is less than or equal to $\dfrac{1}{24\,(10^3)}$

B. The error is less than or equal to $\dfrac{1}{24\,(10^4)}$

C. The error is less than or equal to $\dfrac{1}{8\,(10^3)}$

D. The error is less than or equal to $\dfrac{1}{8\,(10^4)}$

E. The error is less than or equal to $\dfrac{1}{12\,(10^4)}$

23. The first four terms of the Maclaurin series expansion of $(1+x^2)^{\frac{1}{5}}$ are:

A. $(1+x^2)^{\frac{1}{5}} = 1 - \frac{1}{5}x^2 + \frac{4}{2!\,5^2}x^4 - \frac{36}{3!\,5^3}x^6 + \ldots$

B. $(1+x^2)^{\frac{1}{5}} = 1 + \frac{1}{5}x^2 - \frac{8}{2!\,5^2}x^4 + \frac{40}{3!\,5^3}x^6 + \ldots$

C. $(1+x^2)^{\frac{1}{5}} = 1 + \frac{1}{5}x^2 - \frac{4}{2!\,5^2}x^4 + \frac{36}{3!\,5^3}x^6 + \ldots$

D. $(1+x^2)^{\frac{1}{5}} = 1 - \frac{1}{5}x^2 + \frac{8}{2!\,5^2}x^4 - \frac{40}{3!\,5^3}x^6 + \ldots$

E. $(1+x^2)^{\frac{1}{5}} = 1 + \frac{1}{5}x^2 - \frac{3}{2!\,5^2}x^4 + \frac{45}{3!\,5^3}x^6 + \ldots$

24. The Cartesian equation of the polar equation $r = -4\sin\theta$ is

A. $x^2 + y^2 = 4$

B. $(x+2)^2 + (y+2)^2 = 4$

C. $(x+2)^2 + y^2 = 4$

D. $x^2 + (y+2)^2 = 4$

E. $x + (y+2)^2 = 4$

25. The foci of the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$ are

A. $(0, -\sqrt{7})$ and $(0, \sqrt{7})$

B. $(0, -5)$ and $(0, 5)$

C. $(-\sqrt{7}, 0)$ and $(\sqrt{7}), 0$

D. $(-5, 0)$ and $(5, 0)$

E. $(0, -4)$ and $(0, 4)$