MA 16200
Final Exam
05/02/2023
TEST/QUIZ NUMBER:

11

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

You must use a #2 pencil on the scantron answer sheet. Fill in the following on your scantron and blacken the bubbles

1. Your name. If there aren't enough spaces for your name, fill in as much as you can.
2. Section number with a leading zero, e.g. **0032**. (If you don't know your section number, ask your TA.)
3. Test/Quiz number: **11**
4. Student Identification Number: **This is your Purdue ID number with two leading zeros**
5. Blacken in your choice of the correct answer on the scantron answer sheet for questions 1–25.

There are **25** questions, each worth 4 points, for a total of 100 points. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

You may not leave the room before 7:20pm. If you finish the exam between 7:20pm and 8:50pm, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 8:50am, you must remain seated until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam booklet until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, students must put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT SIGNATURE: _____

1. Describe the following two series: $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k\sqrt{k}}$ and $\displaystyle\sum_{k=1}^{\infty} \frac{\cos(k)}{k^3}$

A. One series converges absolutely and one series converges conditionally.

B. One series converges absolutely and one series diverges.

C. One series converges conditionally and one series diverges.

D. Both series converge conditionally.

E. Both series converge absolutely.

2. Evaluate

$$\int_0^{\frac{\pi}{2}} x\cos(x)dx$$

A. $\dfrac{\pi}{2}$

B. 1

C. $\dfrac{3\pi}{2} - 1$

D. $\dfrac{\pi}{2} - 1$

E. $\dfrac{3\pi}{2}$

3. Find the interval of convergence for the following power series:

$$\sum_{k=1}^{\infty} \frac{(x+3)^k}{k2^k}$$

A. $\left(-\frac{7}{2}, -\frac{5}{2}\right]$

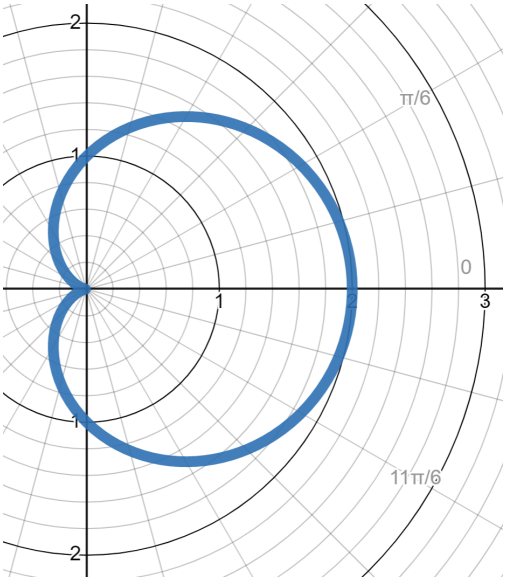
B. $[-5, -1)$

C. $\left[-\frac{7}{2}, -\frac{5}{2}\right]$

D. $(-\infty, \infty)$

E. $(-5, -1)$

4. Which of the following polar equations describes the plot?



A. $r = 2$

B. $r = 1 + \cos(\theta)$

C. $r = 2\cos(\theta)$

D. $r = 1 - 2\cos(\theta)$

E. $\theta = \frac{\pi}{4}$

5. The following function has a partial fraction decomposition:

$$\frac{x^3 - 14x^2 - 3x + 2}{x^4 - x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{x+1}$$

Find C.

A. -2

B. -7

C. 4

D. 3

E. 5

6. Find the length of the curve given by $r = 10\cos(\theta)$ from $\theta = \frac{\pi}{6}$ to $\theta = \frac{\pi}{3}$.

A. $\frac{10\pi}{3}$

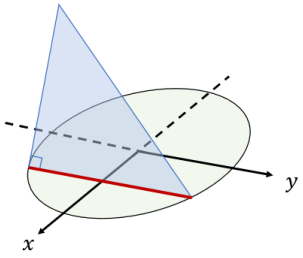
B. $\frac{5\pi}{3}$

C. 10π

D. $\frac{10}{3}$

E. $\frac{5}{3}$

7. Find the volume of the solid whose base is the disk $x^2 + y^2 \leq 9$. The cross sections by planes perpendicular to the x-axis between $x = -3$ and $x = 3$ are isosceles right triangles with one leg in the disk. (Hint: an isosceles right triangle has legs of equal length)



A. 108

B. 72

C. 18

D. 36

E. 144

8. Suppose $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = 5$ and $|\vec{\mathbf{u}} \times \vec{\mathbf{v}}| = 5\sqrt{3}$. Find the angle θ between $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$.

A. $\dfrac{2\pi}{3}$

B. $\dfrac{5\pi}{6}$

C. $\dfrac{\pi}{2}$

D. $\dfrac{\pi}{6}$

E. $\dfrac{\pi}{3}$

9. Using the 2nd order Taylor polynomial of $f(x) = \dfrac{1}{x}$ centered at $a = 3$,

$$\frac{1}{\pi} \approx \frac{1}{3} - \frac{1}{9}(\pi - 3) + \frac{2}{27}(\pi - 3)^2 .$$

According to the Taylor Remainder Theorem, this approximation has error equal to

A. $-\dfrac{(\pi-3)^3}{6c^4}$ for some c between 3 and π

B. $-\dfrac{(\pi-3)^3}{3c^3}$ for some c between 3 and π

C. $-\dfrac{(\pi-3)^3}{3c^4}$ for some c between 3 and π

D. $-\dfrac{(\pi-3)^3}{6c^3}$ for some c between 3 and π

E. $-\dfrac{(\pi-3)^3}{c^4}$ for some c between 3 and π

10. Evaluate

$$\int_0^{\frac{\pi}{3}} \sec^{\frac{5}{2}}(x)\tan(x)dx$$

A. $\dfrac{8\sqrt{2}}{5}$

B. $\dfrac{3\sqrt{2}+1}{5}$

C. $\dfrac{7\sqrt{2}+2}{5}$

D. $\dfrac{8\sqrt{2}-2}{5}$

E. $\dfrac{3\sqrt{2}}{5}$

11. Compute:

$$\sum_{k=1}^{\infty} \frac{(-\pi)^{k-1}}{5^k}$$

A. $\dfrac{\pi}{5+\pi}$

B. $\dfrac{1}{5+\pi}$

C. $\dfrac{5}{5+\pi}$

D. Series diverges

E. $\dfrac{5-\pi}{5+\pi}$

12. A swimming pool has a rectangular base that is 4 m long and 10 m wide. The sides are 2 m high and the pool is half full of water. How much work will it take to empty the pool by pumping the water out over the top of the pool? Write your answer in terms of the gravitational acceleration constant g and the density of water ρ.

A. $60\rho g$ N·m

B. $80\rho g$ N·m

C. $100\rho g$ N·m

D. $40\rho g$ N·m

E. $120\rho g$ N·m

13. Let $f(x) = \frac{2}{3}x^{3/2}$. Find the arc length of $f(x)$ on the interval $[0, 8]$.

A. $\frac{26}{3}$

B. 52

C. 18

D. $\frac{52}{3}$

E. 27

14. The series:

$$\sum_{k=2}^{\infty} \frac{1}{k \ln(k)}$$

A. Diverges by the Integral Test

B. Converges by comparison with $\sum_{k=2}^{\infty} \frac{1}{k^2}$

C. Converges by the Integral Test

D. Diverges by Divergence Test

E. Converges by Divergence Test

15. Consider the series:

$$\sum_{k=1}^{\infty} \tan\left(\frac{1}{k^2}\right)$$

And compare it to the series:

$$\sum_{k=1}^{\infty} b_k = \sum_{k=1}^{\infty} \frac{1}{k^2}$$

What is the result of the Limit Comparison Test?

A. The series diverges because $L > 1$

B. The test is inconclusive because $L = 1$

C. The series diverges because $L = +\infty$

D. The series converges because $0 < L < \infty$

E. The series converges because $L < 0$

16. An appropriate trig substitution will convert the definite integral

$$\int_5^7 \sqrt{x^2 - 6x + 5}\, dx$$

into which of the following integrals?

A. $\displaystyle\int_0^{\frac{\pi}{3}} 4\tan^3(\theta)\sec(\theta)d\theta$

B. $\displaystyle\int_0^{\frac{\pi}{3}} 2\tan(\theta)\sec(\theta)d\theta$

C. $\displaystyle\int_0^{\frac{\pi}{3}} 2\tan(\theta)\sec^3(\theta)d\theta$

D. $\displaystyle\int_0^{\frac{\pi}{3}} 4\tan(\theta)\sec^2(\theta)d\theta$

E. $\displaystyle\int_0^{\frac{\pi}{3}} 4\tan^2(\theta)\sec(\theta)d\theta$

17. The Ratio Test for the series:

$$\sum_{k=1}^{\infty} \frac{(2k)!}{(k!)^2}$$

gives a resulting limit of:

A. $r = 1$ and therefore the test is inconclusive.

B. $r = 0$ and therefore the test is inconclusive.

C. $r > 1$ and therefore the series diverges.

D. $r < \infty$ and therefore the series converges.

E. $r < 1$ and therefore the series converges.

18. How many of the following sequences will converge?

(i) $\left\{\dfrac{n+1}{n-1}\right\}$ (ii) $\left\{\dfrac{(-2)^n}{5^n}\right\}$ (iii) $\left\{\dfrac{n!}{(n+1)!}\right\}$ (iv) $\left\{\dfrac{n!}{2^n}\right\}$

A. 0

B. 3

C. 4

D. 1

E. 2

19. What is the equation in polar form for the horizontal line $y = 3$?

A. $r = 3\sec(\theta)$

B. $r = 3\tan(\theta)$

C. $r = 3\csc(\theta)$

D. $r = 3\cos(\theta)$

E. $r = 3\sin(\theta)$

20. We compute the Taylor series for $f(x) = \dfrac{1}{x^4}$ centered at $a = 7$. What is the coefficient of $(x-7)^3$ in the Taylor series?

A. $-\dfrac{7^4}{6}$

B. $-\dfrac{5}{6 \cdot 7^3}$

C. $-\dfrac{20}{7^7}$

D. $-\dfrac{20}{6 \cdot 7^7}$

E. $-\dfrac{1}{6 \cdot 7^3}$

21. Find $\lim_{x\to 0} \dfrac{9x^6 - 6x^2 + 2\sin(3x^2)}{x^{10}}$

Hint: $\sin(x) = \sum_{k=0}^{\infty} (-1)^k \dfrac{x^{2k+1}}{(2k+1)!}$

A. The limit does not exist

B. 0

C. $\dfrac{3^4}{20}$

D. $\dfrac{3^4}{40}$

E. $\dfrac{3^5}{5}$

22. Let R be the area bounded by the curves $y = \sin(x^2)$, the x-axis, and $x = 0$ and $x = \sqrt{\pi}$. Find the volume of the solid of revolution when R is revolved around the y-axis.

A. 2π

B. 1

C. $\dfrac{\sqrt{\pi}}{2}$

D. $2\sqrt{\pi}$

E. π

23. A pump handle has a pivot at $(0, 0, 0)$ and extends to $P(5, 3, -2)$. A force $\vec{\mathbf{F}} = \langle 2, -1, 1 \rangle$ is applied at P. Find the torque about the pivot.

A. $\langle 5, 10, 0 \rangle$

B. $\langle -1, 1, -1 \rangle$

C. $\langle 1, -9, -11 \rangle$

D. $\langle 5, 11, -1 \rangle$

E. $\langle -5, 9, 1 \rangle$

24. Find the area of the region inside the curve $r = \sqrt{\sin(\theta)}$.

A. 2

B. $\frac{3}{2}$

C. $\frac{1}{2}$

D. 1

E. 0

25. Evaluate

$$\int_0^\infty xe^{-x^2}dx$$

A. The integral is divergent.

B. $\frac{1}{2}$

C. 1

D. $\frac{1}{e}$

E. 0

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