MA 16500
EXAM 1 INSTRUCTIONS
VERSION 01
September 23, 2014

Your name ______________________ Your TA's name ______________________

Student ID # ________________ Section # and recitation time ___________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your question booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the spaces provided for each of the questions 1–12. Do all your work on the question sheets. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

8. There are 12 questions, each worth 8 points. The maximum possible score is
   $8 \times 12 + 4$ (for taking the exam) $= 100$ points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheets and the exam booklets.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets.

## Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of the above rules may result in score of zero.

## Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

## Questions

1. Find the domain of the function

$$\sqrt{\frac{1}{x-2}-\frac{1}{x}}.$$

   A. $(0,2)$
   B. $(-\infty,0)\cup(0,\infty)$
   C. $(-\infty,2)\cup(2,\infty)$
   D. $(-\infty,0)\cup(2,\infty)$
   E. $(-\infty,0)\cup(0,2)\cup(2,\infty)$

**2.** Solve the following equations (a) for $x$ and (b) for $y$.

(a) $\frac{e^{x^2}e^4}{e^{4x}} = 1$

(b) $\log \frac{y+3}{2(y-1)} = 0$

A. (a) 2 (b) $-3$

B. (a) $\log 2$ or $\log 4$ (b) $\log 2$

C. (a) 2 or 4 (b) 0

D. (a) $\log 4$ (b) $\log 2$

E. (a) 2 (b) 5

3. Find a formula for the inverse of the function

$$f(x) = \frac{4x-2}{2x+5}.$$

A. $f^{-1}(x) = \frac{2x+5}{4x-2}$

B. $f^{-1}(x) = \frac{4x-2}{x-5}$

C. $f^{-1}(x) = \frac{-5x+1}{2x-1}$

D. $f^{-1}(x) = \frac{-x-5}{x-2}$

E. $f^{-1}(x) = -\frac{5x+2}{2x-4}$

4. We want to compute the following limit

$$\lim_{x \to 0} x \cdot \sin\left(\frac{1}{x}\right).$$

Choose the right answer with correct reasoning.

A. $\lim_{x\to 0} x \cdot \sin\left(\frac{1}{x}\right) = [\lim_{x\to 0} x] \cdot \left[\lim_{x\to 0} \sin\left(\frac{1}{x}\right)\right]$.
Since $\lim_{x\to 0} x = 0$, the limit we want to compute is also equal to 0.

B. $\lim_{x\to 0} x \cdot \sin\left(\frac{1}{x}\right) = [\lim_{x\to 0} x] \cdot \left[\lim_{x\to 0} \sin\left(\frac{1}{x}\right)\right]$.
Since $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$ does not exist, the limit we want to compute does not exist, either.

C. We observe $x \cdot \sin\left(\frac{1}{x}\right) = \frac{\sin\theta}{\theta}$, setting $\theta = 1/x$.
Since the limit of $\frac{\sin\theta}{\theta}$ is equal to 1, the limit we want to compute is also equal to 1.

D. Since $-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$, we have $-x \leq x \cdot \sin\left(\frac{1}{x}\right) \leq x$.
By applying the Squeeze Theorem, we conclude that the limit we want to compute is equal to 0.

E. Since $-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$, we have $-x \leq x \cdot \sin\left(\frac{1}{x}\right) \leq x$ when $x > 0$ and $-x \geq x \cdot \sin\left(\frac{1}{x}\right) \geq x$ when $x < 0$.
By applying the Squeeze Theorem, we conclude that the limit we want to compute is equal to 0.

5. Compute

$$\lim_{t\to 0}\left(\frac{1}{3}-\frac{1}{\sqrt{4t^2+9}}\right)\frac{1}{t^2}.$$

A. $-2$

B. $\frac{2}{27}$

C. $-3$

D. $\frac{2}{9}$

E. $\frac{1}{54}$

6. How many solutions are there on the interval $[0, 2\pi]$ for the equation $\cos x = \tan x$ ?

A. 0

B. 1

C. 2

D. 3

E. 4

7. We want to find a solution for the equation $x^4 + x - 100 = 0$.

   If we use the Intermediate Value Theorem, in which of the following intervals do we know a solution lies ?

   A. $[0, 1]$
   B. $[1, 2]$
   C. $[2, 3]$
   D. $[3, 4]$
   E. $[4, 5]$

8. How many points of discontinuity do we have on the interval $[-\frac{\pi}{4}, \frac{7\pi}{4}]$ for the function

$$f(x) = \frac{1}{\ln(\cos^2 x)}?$$

A. 0

B. 1

C. 2

D. 3

E. 4

9. Find the values of $a$ and $b$ so that the function

$$f(x) = \begin{cases} x^2 - a & \text{if} \quad x \geq 0 \\ x + 2b & \text{if} \quad 0 < x < 2 \\ 2x - 6 & \text{if} \quad 2 \leq x \end{cases}$$

is continuous on $(-\infty, \infty)$.

A. $a = 4, b = -2$

B. $a = 1, b = -2$

C. $a = 0, b = 0$

D. $a = 1, b = 0$

E. $a = 2, b = -2$

**10.** We know that the slope of the tangent to the graph of a function at point $(3, f(3))$ is 5. Find the exact value of the following limit

$$\lim_{h \to 0} \frac{f(3+h) - f(3-h)}{h}.$$

HINT: The line joining the points $P = (3-h, f(3-h))$ and $Q = (3+h, f(3+h))$ gets closer and closer to the tangent line, and so does its slope. Consider the formula for the slope of the line $PQ$.

A. 0

B. 5

C. 10

D. 15

E. We can not determine the exact value of the limit from the given information.

**11.** The graph of the function $y = f(x) = \frac{3x^2+5}{7x^2-x}$ has

A. 1 vertical and 2 horizontal asymptotes

B. 2 vertical and 1 horizontal asymptotes

C. 1 vertical and 0 horizontal asymptotes

D. 2 vertical and 2 horizontal asymptotes

E. 1 vertical and 1 horizontal asymptotes

12. Find the equation of the tangent line to the curve $y = |x|^3 + 2e^x + 1$ at the point $(0, 3)$.

A. $y = 1$

B. $y = 3|x|^2 + 2e^x$

C. $y = 2x + 3$

D. $y = 2x$

E. The derivative does not exist at $x = 0$, and hence there is no tangent line to the curve at $(0, 3)$.