MA 16500
EXAM 3 INSTRUCTIONS
VERSION 01
November 12, 2024

Your name ______________________ Your TA's name ______________________

Student ID # ________________ Section # and recitation time __________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name, i.e., the name of your recitation instructor (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit **SECTION NUMBER**. Your section number is a 3 digit number. Put 0 at the front to make it a 4 digit number, and then fill it in.

6. **Sign the scantron sheet.**

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–12. While mark all your answers on the scantron sheet, you should show your work on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. There are 12 questions, 8 of which are worth 8 points and 4 of which are worth 10 points. The maximum possible score is

$$8 \textbf{ questions } \times 8 \textbf{ points } + 4 \textbf{ questions } \times 10 \textbf{ points } = 104 \textbf{ points}.$$

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. There is no individual seating. Just follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs/proctors will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor/proctor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

Questions

1. (8 points) Consider the function $f(x) = e^{-x} \cdot \sin x$.

Find the absolute maximum value "**Max**" of $f(x)$ and the absolute minimum value "**Min**" of $f(x)$ on the closed interval $\left[0, \dfrac{3\pi}{2}\right]$.

A. (i) **Max** $= \dfrac{\sqrt{2}}{2e^{\frac{\pi}{4}}}$ (ii) **Min** $= \dfrac{\sqrt{2}}{2e^{\frac{5\pi}{4}}}$

B. (i) **Max** $= \dfrac{\sqrt{2}}{2e^{\frac{\pi}{4}}}$ (ii) **Min** $= -\dfrac{\sqrt{2}}{2e^{\frac{5\pi}{4}}}$

C. (i) **Max** $= \dfrac{\sqrt{2}}{e^{\frac{\pi}{2}}}$ (ii) **Min** $= \dfrac{\sqrt{2}}{2e^{\frac{3\pi}{4}}}$

D. (i) **Max** $= \dfrac{\sqrt{2}}{e^{\frac{\pi}{2}}}$ (ii) **Min** $= -\dfrac{\sqrt{2}}{2e^{\frac{3\pi}{4}}}$

E. (i) **Max** $= \dfrac{\sqrt{3}}{2e^{\frac{2\pi}{3}}}$ (ii) **Min** $= -\dfrac{\sqrt{2}}{2e^{\frac{5\pi}{4}}}$

2. (8 points) Consider the function

$$f(x) = (x+2)^6(x-1)^7$$

on the interval $(-\infty, \infty)$.

Then the function $f(x)$ has a

(i) Local Maximum(s) and (ii) Local Minimum(s).

A. (i) Local Max at $x = -2$ and (ii) Local Min at $x = -8/13$

B. (i) Local Max at $x = -8/13$ and (ii) Local Min at $x = -2$

C. (i) Local Max at $x = -2, 1$ and (ii) Local Min at $x = 8/13$

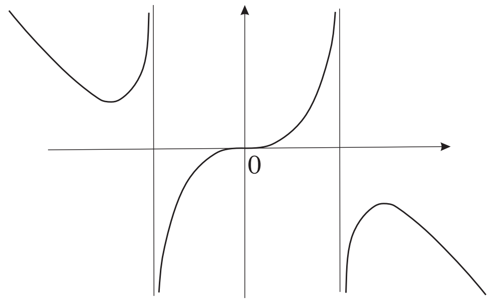
D. (i) Local Max at $x = 1$ and (ii) Local Min at $x = 2, -8/13$

E. (i) Local Max at $x = -8/13, -2$ and (ii) Local Min at $x = 1$

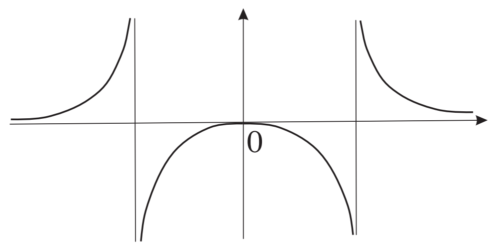
3. (8 points) Which of the following best describes the graph of the function

$$y = f(x) = \frac{x^3}{1 - x^2} \ ?$$

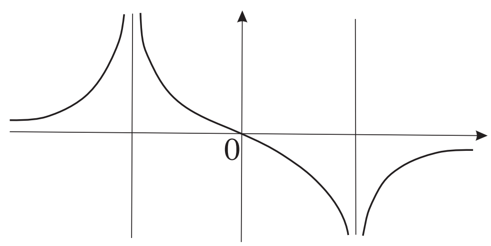
A.



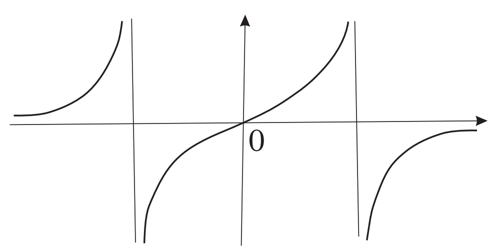
B.



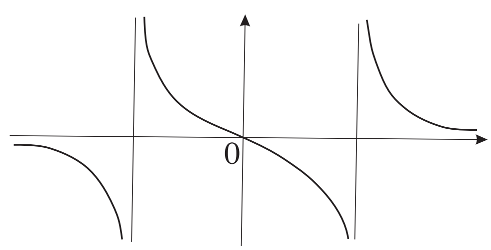
C.



D.



E.



4. (8 points) Compute the following limit

$$\lim_{x\to\infty} \frac{\tan^{-1}(x) - \dfrac{\pi}{2}}{\dfrac{1}{x}}.$$

A. 0

B. 1

C. -1

D. ∞

E. $\dfrac{\pi}{2}$

5. (8 points) Comoute the following limit

$$\lim_{x \to 0^+} (1 + \sin x)^{\frac{1}{2x}}\ .$$

A. e^{12}

B. e^{6}

C. e^{3}

D. $e^{\frac{1}{2}}$

E. $\dfrac{\pi}{6}$

6. (8 points) Compute the following limit

$$\lim_{x\to\infty} 2x \cdot \tan\left(\frac{1}{3x}\right).$$

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{3}{2}$

D. $\frac{2}{3}$

E. ∞

7. (8 points) Determine the exact value for

$$\tan^{-1}\left(-\frac{1}{3}\right) - \tan^{-1}(3).$$

HINT: Consider the function

$$f(x) = \tan^{-1}\left(-\frac{1}{x}\right) - \tan^{-1}(x)$$

on the interval $(0, \infty)$.

What can you observe about its derivative $f'(x)$?

What is the conclusion from this observation ?

A. π

B. $-\pi$

C. $\frac{\pi}{2}$

D. $-\frac{\pi}{2}$

E. 0

8. (8 points) Find the estimate of $\sin(61°)$ using the linear approximation of the function $f(x) = \sin x$ at $a = \pi/3$.

WARNING: $61°$ is measured in terms of degree.

A. $\dfrac{\sqrt{3}}{2} + \dfrac{\pi}{120}$

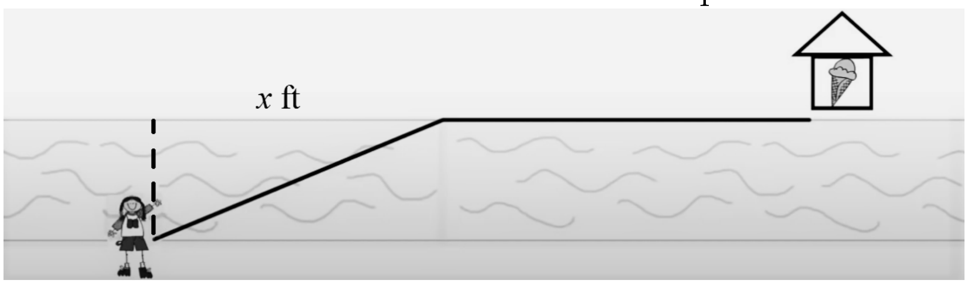
B. $\dfrac{\sqrt{3}}{2} + \dfrac{\pi}{180}$

C. $\dfrac{\sqrt{3}}{2} + \dfrac{\pi}{360}$

D. $\dfrac{1}{2} + \dfrac{\sqrt{3}\pi}{180}$

E. $\dfrac{1}{2} + \dfrac{\sqrt{3}\pi}{360}$

9. (10 points) Liz is standing on the bank of a 50-foot wide river. An ice cream shop is located 100 feet down river on the opposite bank. Liz plans to get to the ice cream shop by a combination of swimming across the river to a point x feet down river on the opposite bank, and then jogging the rest of the way along the bank. She swims at a rate of 4 ft/s and jogs at a rate of 5 ft/s. How many feet downstream of Liz on the opposite bank should she swim to reach the ice cream shop in the least amount of time?



A. 40 ft

B. 50 ft

C. 100 ft

D. $\dfrac{200}{3}$ ft

E. $\dfrac{400}{3}$ ft

10. (10 points) A farmer plans to make four identical and adjacent rectangular pens against a barn, each of which has an area of 100 m^2.

What are the dimensions of each pen that minimize the amount of fence that must be used ?

(Consider the sides as the lengths as x and y with y been the side parallel to the barn).

A. $x = 4\sqrt{5}$ and $y = 5\sqrt{5}$

B. $x = 3\sqrt{5}$ and $y = 4\sqrt{5}$

C. $x = 2\sqrt{3}$ and $y = 5\sqrt{3}$

D. $x = 3\sqrt{3}$ and $y = 4\sqrt{3}$

E. $x = 6$ and $y = 10$

11. (10 points) Find the equation of the line through the point $(4, 3)$ that cuts off the triangle of least area from the first quadrant.

A. $y = -\frac{4}{3}x + \frac{25}{3}$

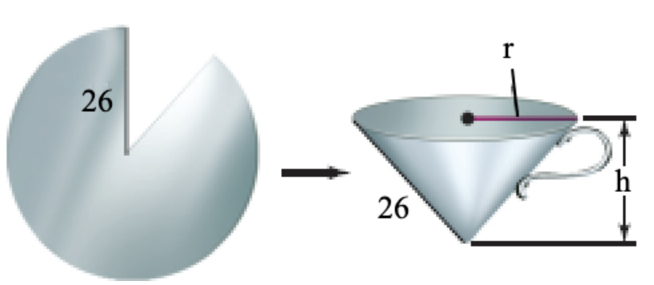
B. $y = -\frac{5}{4}x + 8$

C. $y = -x + 7$

D. $y = -\frac{3}{4}x + 6$

E. $y = -\frac{1}{2}x + 5$

12. (10 points) A cone is constructed by cutting a sector from a circular sheet of metal with radius 26. The cut sheet is then folded up and welded. Find the radius(r) and the height(h) of the cone with maximum volume that can be formed in this way.



A. (i) $r = \dfrac{13\sqrt{6}\pi}{3}$ (ii) $h = \dfrac{13\sqrt{3}\pi}{3}$

B. (i) $r = \dfrac{13\sqrt{6}}{3}$ (ii) $h = \dfrac{13\sqrt{3}}{3}$

C. (i) $r = \dfrac{26\sqrt{6}}{3}$ (ii) $h = \dfrac{26\sqrt{3}}{3}$

D. (i) $r = \dfrac{26\sqrt{3}}{3}$ (ii) $h = \dfrac{26\sqrt{6}}{3}$

E. (i) $r = \dfrac{13\sqrt{6}}{6}$ (ii) $h = \dfrac{13\sqrt{3}}{6}$