NAME ______________________________

10-digit PUID # ______________________________

RECITATION INSTRUCTOR ______________________________

RECITATION TIME ______________________________

LECTURER ______________________________

INSTRUCTIONS

1. There are 9 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. Also write your name at the top of pages 2–9.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet. No partial credit will be given, but if you show your work on the test booklet, it may be used in borderline cases.
4. No books, notes, calculators, or any electronic devices may be used on this exam.
5. Each problem is worth 8 points. The maximum possible score is 200 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION NUMBER, put 0 in the first column and then enter the 3-digit section number. For example, for section 016 write 0016, and fill in the little circles.
   (c) On the bottom, under TEST/QUIZ NUMBER, write 01 and fill in the little circles.
   (d) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your 10-digit PUID, and fill in the little circles.
   (e) Using a #2 pencil, put your answers to questions 1–25 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your recitation instructor.

1. The domain of $f(x) = \sqrt{1 - \ln x}$ is

A. $x \le e$

B. $0 < x \le e$

C. $e < x$

D. $0 < x \le 1$

E. $1 \le x \le e$

2. If $f(x) = x + \dfrac{1}{x}$, then $\displaystyle\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} =$

A. $\dfrac{1}{2}$

B. $\dfrac{5}{2}$

C. $\dfrac{1}{3}$

D. $\dfrac{3}{4}$

E. Does not exist

3. $\displaystyle\lim_{x \to 0} \frac{x - x^2}{|x|} =$

A. 1

B. -1

C. 0

D. -2

E. Does not exist

4. Find an equation for the line tangent to the curve $y = x^2 \ln(x^2)$ at the point where $x = e$.

A. $y - 3ex + e^2 = 0$

B. $y - 6ex + 4e^2 = 0$

C. $y + ex - 3e^2 = 0$

D. $y - 3ex + 2e^2 = 0$

E. $y + 6ex + 2e^2 = 0$

5. If $f'(x) = g'(x)$ for all x in $(0, \infty)$ and $f(1) - g(1) = 1$, then $f(5) - g(5) =$

A. 5

B. -1

C. 1

D. -5

E. cannot be determined

6. If $y = \tan^{-1}(x^2)$, then $\dfrac{d^2y}{dx^2} =$

A. $\dfrac{2x}{1+x^4}$

B. $\dfrac{6x^4 - 2}{(1+x^4)^2}$

C. $\dfrac{-4}{(1+x^4)^2}$

D. $\dfrac{2 - 6x^4}{(1+x^4)^2}$

E. $\dfrac{8x^4}{(1+x^4)^2}$

7. Find the slope of the tangent line to the curve $\cos x \sin y = \dfrac{1}{2\sqrt{2}}$ at the point $(\frac{\pi}{3}, \frac{\pi}{4})$.

A. $\dfrac{1}{\sqrt{3}}$

B. $\sqrt{3}$

C. $-\sqrt{3}$

D. $-\dfrac{1}{\sqrt{3}}$

E. $\dfrac{2}{\sqrt{3}}$

8. Estimate $\sqrt{36.3}$ using a linear approximation at $a = 36$.

A. 6.010

B. 6.015

C. 6.020

D. 6.025

E. 6.030

9. Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x$ on the interval $[0, 2]$.

A. $\max 20,\ \min -3$

B. $\max 3,\ \min 1$

C. $\max 2,\ \min 0$

D. $\max 0,\ \min -7$

E. $\max 4,\ \min -7$

10. A box with square base and open top must have a volume of 4 cubic meters. Find the height of the box that has the smallest possible area.

A. $h = \frac{3}{2}$

B. $h = 1$

C. $h = 4$

D. $h = \frac{1}{2}$

E. $h = \frac{1}{4}$

11. The half–life of a certain radioactive substance is 10 years. How long will it take for 18 gms of the substance to decay to 6 gms?

A. $6 \ln 10$ years

B. $10 \ln 6$ years

C. $10 \dfrac{\ln 3}{\ln 6}$ years

D. $18 \ln 10$ years

E. $10 \dfrac{\ln 3}{\ln 2}$ years

12. The graph of the function $f(x) = 3x^5 - 5x^4$ has inflection points when $x =$

A. 0 and 1

B. 0 and 2

C. 1

D. 2

E. never

13. If $f(x) = x^5 - 5x + 3$, which one of the following statements is true?

A. $x = 1$ is the only critical number of f, and f has a local max. at 1

B. $x = -1$ is the only critical number of f, and f has a local min. at -1

C. $x = 1, -1$ are the only critical numbers of f, and f has a local max. at -1 and a local min. at 1.

D. $x = 1, -1$ are the only critical numbers of f, and f has a local min. at -1 and a local max. at 1.

E. $x = 1, -1$ are the only critical numbers of f, but f has neither a local max, nor a local min. at these critical numbers.

14. $\dfrac{d}{dx} \displaystyle\int_0^{x^3} \frac{t}{\sqrt{1+t^3}}\, dt =$

A. $\dfrac{3x^5}{\sqrt{1+x^9}}$

B. $\dfrac{x^3}{\sqrt{1+x^9}}$

C. $\dfrac{3x^3}{\sqrt{1+x^3}}$

D. $\dfrac{5x^3}{\sqrt{1+x^6}}$

E. $\dfrac{3x^5}{\sqrt{1+x^3}}$

15. Two sides of a triangle have fixed lengths of 4 meters and 5 meters, while the angle θ between them is increasing at a rate of 0.06 rad/sec. Find the rate at which the area of the triangle is increasing when $\theta = \dfrac{\pi}{3}$.

A. 0.6 m^2/sec

B. 0.3 m^2/sec

C. $0.3\sqrt{3}$ m^2/sec

D. $0.2\sqrt{3}$ m^2/sec

E. $0.6\sqrt{3}$ m^2/sec

16. $\displaystyle\int_0^1 \frac{\tan^{-1} x}{1+x^2}\, dx =$

A. $\frac{\pi}{4}$

B. $\frac{\pi^2}{32}$

C. $\frac{\pi^2}{8}$

D. $\frac{\pi}{2}$

E. $\frac{\pi^2}{16}$

17. $\displaystyle\int_1^e \frac{(\ln x)^3}{x}\, dx =$

A. $\frac{1}{3}$

B. $\frac{e}{4}$

C. e

D. e^2

E. $\frac{1}{4}$

18. Find the area of the region between the graph of $y = \dfrac{3}{\sqrt{1-x^2}}$ and the x–axis, from $x = -\dfrac{\sqrt{2}}{2}$ to $x = \dfrac{\sqrt{2}}{2}$.

A. π

B. $\frac{\pi}{2}$

C. $\frac{5\pi}{2}$

D. $\frac{3\pi}{2}$

E. $\frac{2\pi}{3}$

19. $\displaystyle\int_{-e^2}^{-e} \frac{3}{x}\,dx =$

A. $e^3 - e^6$

B. -3

C. 3

D. $\dfrac{3}{e^4} - \dfrac{3}{e^2}$

E. $e^6 - e^3$

20. A particle moves in a straight line and its acceleration is given by $a(t) = 6t + 4$. Its initial position is $s(0) = 9$ and its position when $t = 1$ is $s(1) = 6$. Find the velocity of the particle when $t = 2$.

A. $v(2) = 14$

B. $v(2) = 7$

C. $v(2) = 4$

D. $v(2) = 9$

E. $v(2) = 0$

21. If $y = x^{\ln x}$, find $\dfrac{dy}{dx}$ at $x = e$.

A. 1

B. 2

C. 3

D. e

E. 0

22. $\lim_{x\to 0} \dfrac{\sin x - x}{x^3} =$

A. $\frac{1}{6}$

B. $-\frac{1}{6}$

C. $\frac{1}{2}$

D. $\frac{1}{3}$

E. 0

23. The equation $x^3 + 1 = x$ has exactly one root in the interval $(-3, 2)$. This root is in the interval.

A. $(-3, -2)$

B. $(-2, -1)$

C. $(-1, 0)$

D. $(0, 1)$

E. $(1, 2)$

24. The focus of the parabola $y^2 - 2y - 8x - 23 = 0$ is at the point

A. $(-1, -3)$

B. $(1, -1)$

C. $(-1, 1)$

D. $(3, 1)$

E. $(1, 2)$

25. Find an equation of the ellipse with vertices $(\pm 3, 0)$ and foci $(\pm 1, 0)$.

A. $4x^2 + 36y^2 = 144$

B. $8x^2 + 9y^2 = 72$

C. $x^2 + 9y^2 = 9$

D. $9x^2 + y^2 = 9$

E. $9x^2 + 8y^2 = 72$