MA 16500
FINAL EXAM INSTRUCTIONS
VERSION 01
December 13, 2017

Your name ____________________ Your TA's name ____________________

Student ID # _______________ Section # and recitation time __________

1. You must use a <u>#2 pencil</u> on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. <u>Write 01</u> in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **<u>TA's name (NOT the lecturer's name)</u>** and the <u>course number</u>.

4. Fill in your <u>NAME</u> and <u>PURDUE ID NUMBER</u>, and blacken in the appropriate spaces.

5. Fill in the four-digit **<u>SECTION NUMBER</u>**.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–25. **All the answers must be marked on the scantron sheet.** In case what is marked on the scantron sheet is different from what is marked on the exam booklet, we compute the final score based upon what is marked on the scantron sheet.

8. While marking all your answers on the scantron sheet, you should **<u>show your work on the exam booklet</u>**. In case of a suspicious activity of academic dishonesty and/or under certain circumstances, we require that the correct answer on the scantron sheet must be supported by the work on the exam booklet.

9. There are 25 questions, each worth 8 points. The maximum possible score is $8 \times 25 = 200$ points.

10. <u>NO calculators, electronic device, books, or papers are allowed.</u> Use the back of the test pages for scrap paper.

11. After you finish the exam, <u>turn in BOTH the scantron sheet and the exam booklet</u>.

12. If you finish the exam before 12:25, you may leave the room after turning in the scantron sheets and the exam booklets. <u>If you don't finish before 12:25, you should REMAIN SEATED</u> until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

Questions

1. Find the domain of the function

$$f(x) = \frac{1}{\ln(x^2 - 4)}.$$

A. $(-2,\ 2)$

B. $(-\infty,\ -2) \cup (2,\ \infty)$

C. $(-\infty,\ -2] \cup [2,\ \infty)$

D. $(-\infty,\ \infty)$

E. $(-\infty,\ -\sqrt{5}) \cup (-\sqrt{5},\ -2) \cup (2,\ \sqrt{5}) \cup (\sqrt{5},\ \infty)$

2. We have the information

$$\sin\theta = \frac{12}{13} \quad \text{and} \quad \frac{\pi}{2} < \theta < \pi.$$

Determine the value of $\cot\theta$.

A. $\dfrac{5}{12}$

B. $-\dfrac{5}{12}$

C. $\dfrac{12}{5}$

D. $-\dfrac{12}{5}$

E. $-\dfrac{5}{13}$

3. How many solutions are there on the interval $[0, 2\pi]$ for the equation

$$\cos^2 x - \sin^2 x = 0 \ ?$$

A. 0

B. 1

C. 2

D. 3

E. 4

4. Find a formula for the inverse of the function

$$f(x) = \frac{1 - e^x}{1 + e^x}.$$

A. $f^{-1}(x) = \dfrac{x-1}{x+1}$

B. $f^{-1}(x) = \dfrac{1+e^x}{1-e^x}$

C. $f^{-1}(x) = \ln\left(\dfrac{1+x}{1-x}\right)$

D. $f^{-1}(x) = \ln\left(\dfrac{x-1}{x+1}\right)$

E. $f^{-1}(x) = \ln\left(\dfrac{1-x}{1+x}\right)$

5. Find the values of a and b so that the function

$$f(x) = \begin{cases} \dfrac{x^2+4x-a}{x^2+2x-3} & \text{if } \ x<1 \\ x-b & \text{if } \ x \geq 1. \end{cases}$$

is continuous on $(-\infty, \infty)$.

A. $a = 12, \ b = -1$

B. $a = 12, \ b = 1$

C. $a = 5, \ b = -1/2$

D. $a = 5, \ b = 1/2$

E. No matter how we choose the values for a and b, the function $f(x)$ can never be continuous on $(-\infty, \infty)$.

6. Suppose we have a function $f(x)$ such that $f'(1) = 3$.

Then the value of the following limit

$$\lim_{h \to 0} \frac{f(1+2h) - f(1-h)}{7h}$$

is:

A. 3

B. $\frac{2}{7}$

C. $\frac{3}{7}$

D. $\frac{6}{7}$

E. $\frac{9}{7}$

7. Given the curve $y = x\sqrt{x}$, find the equation of the tangent line that is parallel to the line $y = 2 + x$.

A. $y = x - \dfrac{4}{9}$

B. $y = x - \dfrac{8}{27}$

C. $y = x + \dfrac{8}{27}$

D. $y = x - \dfrac{4}{27}$

E. $y = x + \dfrac{4}{27}$

8. Consider the function $f(x) = \tan(x)^{\cos(x)}$.

Compute $f'\left(\dfrac{\pi}{4}\right)$.

A. 0

B. 1

C. 2

D. $\sqrt{2}$

E. $\ln 2$

9. Find the formula for $\cos(2\sin^{-1} x)$.

A. $1 - 2x^2$

B. x^2

C. $\dfrac{x}{\sqrt{1+x^2}}$

D. $\dfrac{1-x^2}{\sqrt{1+x^2}}$

E. $2x\sqrt{1-x^2}$

HINT:

- $\sin 2\theta = 2\sin\theta\cos\theta$
- $\cos 2\theta = 1 - 2\sin^2\theta$

10. Suppose that $F(x) = f(x)f(g(x))$ and that the functions f and g satisfy the following conditions:

$$\begin{cases} f(1) = 5, & f(2) = 3, & f(3) = -1 \\ f'(1) = 4, & f'(2) = 3, & f'(3) = -2 \\ g(1) = 3, & g(2) = 2, & g(3) = -1 \\ g'(1) = 2, & g'(2) = 3, & g'(3) = 4 \end{cases}$$

Find $F'(1)$.

A. 24

B. -24

C. 27

D. 32

E. -16

11. Find the slope of the tangent to the curve given by the equation

$$x^2 + 2xy - y^2 + x = 2$$

at the point $(x, y) = (1,\ 2)$.

A. $-3/2$

B. $7/2$

C. $9/4$

D. $7/4$

E. $9/2$

12. If we use the linear approximation for $f(x) = \sqrt{x}$ at $a = 25$, then the estimate for $\sqrt{24.8}$ is:

A. 4.94

B. 4.96

C. 4.98

D. 4.99

E. 5.00

13. Find the exact values for

(a) $\sin^{-1}\left(\sin\left(\dfrac{3\pi}{5}\right)\right)$

(b) $\sin\left(2\sin^{-1}\left(\dfrac{5}{13}\right)\right)$

A. (a) $\dfrac{3\pi}{5}$ (b) $\dfrac{119}{169}$

B. (a) $\dfrac{3\pi}{5}$ (b) $\dfrac{120}{169}$

C. (a) $\dfrac{2\pi}{5}$ (b) $\dfrac{119}{169}$

D. (a) $\dfrac{2\pi}{5}$ (b) $\dfrac{120}{169}$

E. (a) $-\dfrac{2\pi}{5}$ (b) $-\dfrac{120}{119}$

14. A man walks along a straight path at a speed of 6 ft/sec.

A search light is located on the ground 20 ft from the path and is kept focused on the man.

At what rate is the search light rotating when the man is 10 ft from the point on the path closest to the search light ?

A. $\dfrac{\pi}{4}$ rad/s

B. $\dfrac{\pi}{7}$ rad/s

C. $\dfrac{6}{25}$ rad/s

D. $\dfrac{8}{25}$ rad/s

E. $\dfrac{10}{25}$ rad/s

15. Car A is traveling west at 60 mi/h and car B is traveling north at 65 mi/h. Both are headed for the intersection of the two roads.

At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection ?

A. 0.5 mi/h

B. 0.7 mi/h

C. 64 mi/h

D. 78 mi/h

E. 88 mi/h

16. Compute the following limits:

(a) $\lim_{x\to\infty} \sin(x) \cdot \tan\left(\frac{3}{x}\right)$

(b) $\lim_{x\to 1^+} \left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$

A. (a) 0 (b) 0

B. (a) DNE (b) 0

C. (a) 3 (b) ∞

D. (a) 3 (b) 1/2

E. (a) 0 (b) 1/2

17. Evaluate the integration

$$\int_0^1 x^3 \sqrt{1+x^2}\, dx.$$

A. $\dfrac{2}{15}\left(1+\sqrt{2}\right)$

B. $\dfrac{4}{15}\left(1+\sqrt{2}\right)$

C. $\dfrac{1}{5}\left(4\sqrt{2}-1\right)$

D. $\dfrac{2}{5}\left(4\sqrt{2}-1\right)$

E. $-\dfrac{2}{15}$

18. Compute

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(3 + i \cdot \frac{5}{n}\right)^2 \cdot \frac{5}{n}$$

A. $\dfrac{55}{2}$

B. 55

C. $\dfrac{\pi}{4}$

D. $\dfrac{485}{3}$

E. ∞

19. Compute the following limits:

(a) $\lim_{x\to 0^+}(1+\sin 4x)^{\cot x}$

(b) $\lim_{x\to 0} \dfrac{\tan x - x}{x^3}$

A. (a) ∞ (b) DNE

B. (a) 1 (b) 0

C. (a) 4 (b) 1

D. (a) e^4 (b) 0

E. (a) e^4 (b) $\dfrac{1}{3}$

20. The derivative of the function f is given by

$$f'(x) = (x-1)x(x+1)^2(x+2)$$

The function f has a local minimum only at

A. $x=-2$ and $x=1$

B. $x=-2$

C. $x=-2$ and $x=0$

D. $x=-1$ and $x=0$

E. $x=-1$ and $x=1$

21. Choose the one which describes best the graph of the function

$$f(x) = \frac{x}{x^2 - 4}$$

over its domain.

A.

B.

C.

D.

E.

22. The number of bacteria in a cell culture is initially observed to be 100. Two hours later the number is 200. Assuming that the bacteria grow exponentially, how many hours after the initial observation does the number of bacterial become equal to 500?

A. $\dfrac{\ln 5}{\ln 2}$

B. $\dfrac{\ln 2}{\ln 5}$

C. $\dfrac{2(\ln 5 + \ln 2)}{\ln 3 + \ln 2}$

D. $\dfrac{2 \ln 2}{\ln 5}$

E. $\dfrac{2(\ln 5)}{\ln 2}$

23. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 4 cm and 5 cm if two sides of the rectangle lie along the legs.

A. 2

B. $\dfrac{5}{2}$

C. 3

D. 5

E. 6

24. Find the focus of the parabola given by the equation

$$y^2 + 4y + 4x - 20 = 0.$$

A. $(6, -2)$

B. $(5, -2)$

C. $(3, -2)$

D. $(-7, 2)$

E. $(-5, -2)$

25. Find an equation of the hyperbola that has the vertices $(-9, 1), (3, 1)$ and foci $(-10, 1), (4, 1)$.

A. $\dfrac{(x+3)^2}{36} - \dfrac{(y-1)^2}{13} = 1$

B. $\dfrac{(x-3)^2}{13} - \dfrac{(y+1)^2}{36} = 1$

C. $\dfrac{(y-1)^2}{36} - \dfrac{(x+3)^2}{13} = 1$

D. $\dfrac{(y-1)^2}{13} - \dfrac{(x+3)^2}{36} = 1$

E. $\dfrac{x^2}{36} - \dfrac{y^2}{13} = 1$