MA 16500
FINAL EXAM INSTRUCTIONS
VERSION 01
December 11, 2019

Your name ______________________ Your TA's name ______________________

Student ID # ________________ Section # and recitation time ___________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit **SECTION NUMBER**.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–25. **All the answers must be marked on the scantron sheet.** In case what is marked on the scantron sheet is different from what is marked on the exam booklet, we compute the final score based upon what is marked on the scantron sheet.

8. While marking all your answers on the scantron sheet, you should **show your work on the exam booklet**. In case of a suspicious activity of academic dishonesty and/or under certain circumstances, we require that the correct answer on the scantron sheet must be supported by the work on the exam booklet.

9. There are 25 questions, each worth 8 points. The maximum possible score is $8 \times 25 = 200$ points.

10. **NO calculators, electronic device, books, or notes are allowed.** Use the back of the test pages for scrap paper.

11. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

12. If you finish the exam before 5:25 PM, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 5:25 PM, you should REMAIN SEATED until your TA/Proctor comes and collects your scantron sheet and exam booklet. Do NOT form a line in front of your TA while waiting to turn in the scantron sheet and exam booklet. Leaving your seat before submitting the scantron sheet and exam booklet might be regarded as an act of academic dishonesty.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

Questions

1. Consider the function

$$f(x) = \frac{e^x - 1}{e^x + 1}.$$

Find

(i) the formula for the inverse function $f^{-1}(x)$

(ii) the domain of the inverse function $f^{-1}(x)$.

A. (i) $\ln \frac{1+x}{1-x}$ (ii) $(-\infty, 1) \cup (1, \infty)$

B. (i) $\ln \frac{1+x}{1-x}$ (ii) $(-1, 1)$

C. (i) $\ln \frac{1-x}{1+x}$ (ii) $(-\infty, 1) \cup (1, \infty)$

D. (i) $\ln \frac{1-x}{1+x}$ (ii) $(-1, 1)$

E. (i) $\frac{1+x}{1-x}$ (ii) $(-\infty, 1) \cup (1, \infty)$

2. Find the values of a and b so that the function

$$f(x) = \begin{cases} x^2 - b & \text{if } x \leq 1 \\ \dfrac{x^2 + 6x - a}{x^2 + 2x - 3} & \text{if } 1 < x \end{cases}$$

is continuous on $(-\infty, \infty)$.

A. $a = -7/2, b = 15$

B. $a = 7, b = 1$

C. $a = 7, b = -1$

D. $a = -7, b = 1$

E. $a = 16, b = -5$

3. Compute the following limits.

(i) $\lim_{x\to 2} \dfrac{x^2-4}{x^2-5x+6}$

(ii) $\lim_{x\to 0} x\sin\left(\dfrac{1}{x}\right)$

A. (i) -4 (ii) 0

B. (i) -4 (ii) 1

C. (i) DNE (ii) 1

D. (i) 4 (ii) 1

E. (i) -4 (ii) DNE

4. Compute the following limits.

(i) $\lim_{x\to 0} (1+2x)^{1/x}$

(ii) $\lim_{x\to\infty} \left(\sqrt{x^2+4x+5}-x\right)$

A. (i) e^{-2} (ii) 0

B. (i) 1 (ii) 0

C. (i) e^2 (ii) 1

D. (i) e^2 (ii) 2

E. (i) e^2 (ii) 0

5. The absolute minimum "min" and the absolute maximum "MAX" of the function $f(x) = (1-x)e^x$ on the interval $[-1, 1]$ are:

A. $\min = 0, \mathrm{MAX} = 1.$

B. $\min = 0, \mathrm{MAX} = e.$

C. $\min = 2/e, \mathrm{MAX} = 1.$

D. $\min = 2/e, \mathrm{MAX} = e.$

E. $\min = 0, \mathrm{MAX} = 2/e.$

6. Consider the curve defined by the equation

$$xe^y - ye^x = e^2 - 2e.$$

Compute $\dfrac{dy}{dx}$ at the point $(1, 2)$.

A. $\dfrac{e}{2-e}$

B. $\dfrac{2-e}{e}$

C. $\dfrac{e-1}{2-e}$

D. $\dfrac{2-e}{e-1}$

E. $\dfrac{e}{e-1}$

7. If we use the linear approximation for $f(x) = \sqrt[3]{x}$ at $a = 27$, then which of the following best describes the estimate for $\sqrt[3]{24}$?

A. $2 + \dfrac{26}{27}$

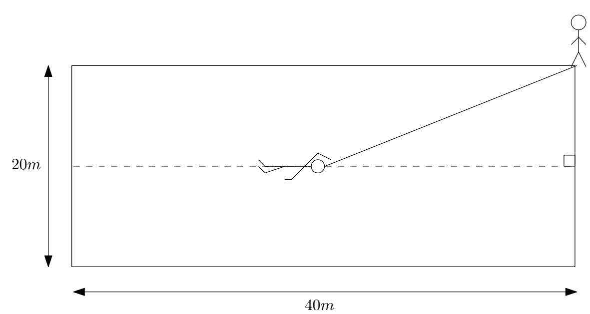
B. $2 + \dfrac{25}{27}$

C. $2 + \dfrac{8}{9}$

D. $3 + \dfrac{1}{27}$

E. $3 + \dfrac{2}{27}$

8. Fred is swimming a race in a pool 40 m long and 20 m wide. He is swimming his race in the center lane so that he is always 10m from the either side. His coach is standing at the corner of the pool by the finish line. If Fred swims at a constant rate of 2 m/s, what is the rate at which the distance between Fred and his coach is decreasing when he is halfway through the race.



A. $\dfrac{\sqrt{5}}{4}$ m/s

B. $\sqrt{5}$ m/s

C. $\dfrac{\sqrt{5}}{2}$ m/s

D. $\dfrac{2}{\sqrt{5}}$ m/s

E. $\dfrac{4}{\sqrt{5}}$ m/s

9. A 10-foot plank of wood is leaning against a vertical wall and its bottom is being pushed toward the wall at the rate of 2 ft/sec.

At what rate is the angle θ between the plank and the ground changing when the acute angle the plank makes with the ground is $\pi/4$?

A. $\dfrac{2\sqrt{2}}{15}$ rad/sec

B. $\dfrac{2}{5}$ rad/sec

C. $\sqrt{2}$ rad/sec

D. $\dfrac{\sqrt{2}}{5}$ rad/sec

E. $\pi/3$ rad/sec

10. Which of the following statements are ALWAYS true for a function which is differentiable over $(-\infty, \infty)$?

1. If $f'(x) < 0$ for $x < 0$ and $f'(0) = 0$, then f has a local minimum at $x = 0$.

2. If f has a local maximum at $x = 0$, then $f'(0) = 0$.

3. If $f'(x) < 0$ for $x < 0$ and $f'(x) > 0$ for $x > 0$, then f has an absolute minimum at $x = 0$.

A. ALL 1,2,3, are true.

B. ONLY 1,2, are true.

C. ONLY 1,3, are true.

D. ONLY 2,3, are true.

E. NONE are true.

11. What is the area of the largest rectangle inscribed in the upper half of the ellipse defined by the equation $\dfrac{x^2}{5^2} + \dfrac{y^2}{2^2} = 1$?

A. 10

B. 50

C. $10\sqrt{2}$

D. $5^2 \cdot 2^2$

E. $5/2$

12. Compute the following limit

$$\lim_{n\to\infty} \left(\sum_{i=1}^{n} \frac{n}{n^2 + i^2} \right)$$

HINT: The above limit can be considered as the Riemann sum expression for a certain integration. Use the equation

$$\sum_{i=1}^{n} \frac{n}{n^2 + i^2} = \sum_{i=1}^{n} \frac{1}{1 + \left(\frac{i}{n}\right)^2} \cdot \frac{1}{n}$$

if necessary.

A. 1

B. $\pi/4$

C. $\pi/3$

D. $\pi/2$

E. $\dfrac{1+\sqrt{2}}{2}$

13. Initially there was 10 gm of a radioactive substance. After 3 years, only 3 gm reamined. What is the half-life of this substance ?

A. $\dfrac{\ln(10/3)}{4}$ years.

B. $\dfrac{3\ln 2}{\ln(10/3)}$ years.

C. $2\ln 2$ years.

D. $\dfrac{10\ln 2}{3}$ years.

E. $\dfrac{3}{5\ln(10/3)}$ years.

14. How many solutions are there on the interval $[0, 2\pi]$ for the following equation

$$\cos(2x) = \sin x \text{ ?}$$

A. 0

B. 1

C. 2

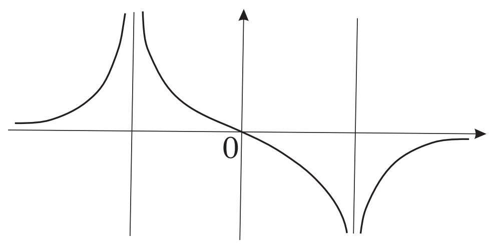
D. 3

E. 4

15. Which of the following best describes the graph of the function

$$y = f(x) = \frac{x}{x^2 - 1}\ ?$$

A.



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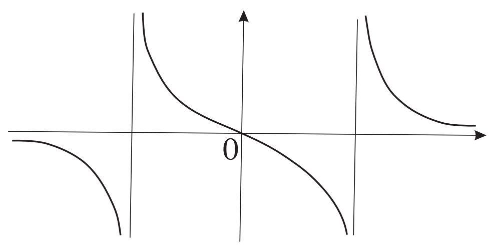

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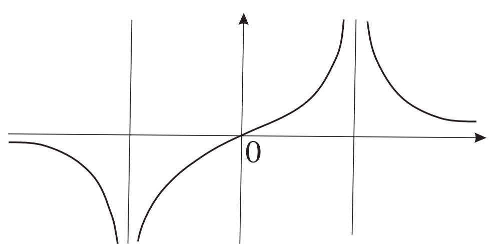

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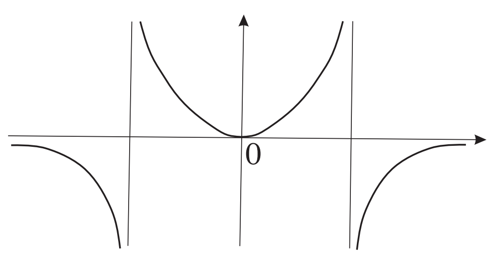
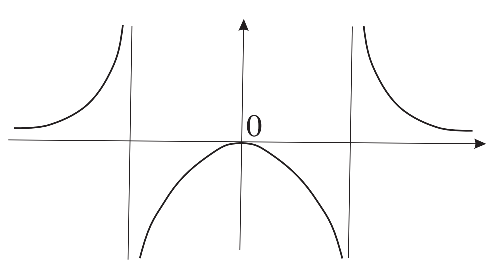


E.



16. Suppose that $F(x) = f^{-1}\left(g(x)^3\right)$ and that the functions f (which is one-to-one, and hence has its inverse) and g satisfy the following conditions. Find $F'(1)$.

$$\begin{cases} f(3) = 8, & f(8) = 5, \\ f'(1) = 4, & f'(8) = 3, \quad f'(3) = -2 \\ g(1) = 2, & g'(1) = 2 \end{cases}$$

A. $8/3$

B. 4

C. -6

D. 8

E. -12

17. We would like to compute the following limit

$$L = \lim_{h \to 0} \frac{(3+2h)^{5+2h} - 3^5}{h}$$

in terms of the derivative of a function $f(x)$.

Choose the right statement.

A. $L = f'(5)$ where $f(x) = 3^x$

B. $L = 2f'(5)$ where $f(x) = 3^x$

C. $L = f'(0)$ where $f(x) = (3+x)^{5+x}$

D. $L = 2f'(0)$ where $f(x) = (3+x)^{5+x}$

E. $L = 2f'(0)$ where $f(x) = (3+x)^5$

18. Set

$$F(x) = \int_x^{x^2} \sqrt{\ln t}\; dt.$$

Find $F'(e)$.

A. $\sqrt{2} - 1$

B. $e\sqrt{2} - 1$

C. $2e\sqrt{2} - 1$

D. $e^2 - e$

E. $\sqrt{e} - 1$

19. Evaluate the following two integrals

(i) $\displaystyle\int_0^{\pi/4} \tan^3 x \sec^2 x \; dx$

(ii) $\displaystyle\int_{-\pi/4}^{\pi/4} \tan^3 x \sec^2 x \; dx$

A. (i) $1/4$ (ii) 0

B. (ii) $1/3$ (ii) 0

C. (i) $1/4$ (ii) $1/2$

D. (i) 0 (ii) $2/3$

E. (i) 0 (ii) $1/4$

20. What value(s) of b minimizes the integral

$$\int_2^b x^2(x-7)\,dx\ ?$$

A. $b = 7$

B. $b = 0$

C. $b = 0, 7$

D. $b = 7/2$

E. $b = 7$ is the value of b which maximizes the integral, but there is no value of b which minimizes the integral.

21. We have a function whose first derivative is given by the formula

$$f'(x) = (x-1)^3(x+1)^3.$$

Find

(i) the x-coordinate(s) of the local maximum (maxima)

(ii) the x-coordinate(s) of the inflection point(s).

A. (i) $-1, 1$ (ii) $-1, 0, 1$

B. (i) 1 (ii) $-1, 0, 1$

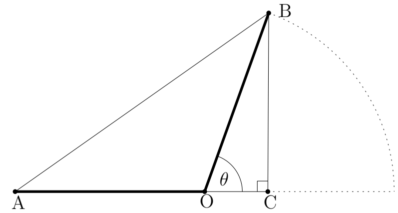
C. (i) -1 (ii) $-1, 0, 1$

D. (i) -1 (ii) $-1, 1$

E. (i) -1 (ii) 0

22. Two wooden bars of equal length AO = BO = 4 ft are connected by a hinge at point O so that one can rotate the bar BO around as shown in the picture below.

Find the maximum area of the triangle △ABC when $0 < \theta < \pi$.



A. $3\sqrt{3}/8$

B. $\sqrt{3}$

C. $3\sqrt{3}$

D. $6\sqrt{3}$

E. $\pi/3$

23. Suppose that the function f is continuous on the interval $[0, 5]$ and diffrentiable on $(0, 5)$. We have $f(2) = 4$ and $f(4) = 6$.

Applying the Mean Value Theorem, we can conclude that there is a value $c \in (0, 5)$ such that $f'(c)$ is equal to:

A. $f'(c) = 2/5$

B. $f'(c) = 1/2$

C. $f'(c) = 1$

D. $f'(c) = 5/2$

E. We cannot conclude anything using the Mean Value Theorem, since the given values of f are not on the end points 0 and 5.

24. Consider the graph of the following function

$$y = f(x) = \frac{x-5}{\sqrt{x^2 - 5x}}.$$

Find the number of

(i) the vertical asymptote(s),

(ii) the horizontal asymptote(s).

A. (i) 2 (ii) 1

B. (i) 1 (ii) 1

C. (i) 1 (ii) 2

D. (i) 0 (ii) 1

E. (i) 2 (ii) 2

25. Evaluate the following limit

$$\lim_{x \to \pi} \frac{e^{\sin x} - 1}{x - \pi}.$$

A. 1

B. -1

C. 0

D. ∞

E. e^{π}

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