

NAME \_\_\_\_\_

10-DIGIT PUID \_\_\_\_\_

RECITATION INSTRUCTOR \_\_\_\_\_

RECITATION TIME \_\_\_\_\_

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TOTAL	/100

## DIRECTIONS

- Write your name, 10-digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, and 4.
- The test has four (4) pages, including this one.
- Write your answers in the boxes provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses in the left hand margin.
- No books, notes or calculators may be used on this test.

- (10) 1. Let  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  be three-dimensional vectors. For each statement below, circle T if the statement is always true, or F if it is not always true.

(i)  $(2\vec{a}) \cdot (3\vec{b}) = 6\vec{a} \cdot \vec{b}$  T F

(ii)  $(\vec{a} \cdot \vec{b})\vec{c} = (\vec{b} \cdot \vec{c})\vec{a}$  T F

(iii)  $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$  T F

(iv)  $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$  T F

(v) If  $\vec{a} \times \vec{b} = \vec{0}$  then  $\vec{a}$  and  $\vec{b}$  are parallel T F

- (4) 2. If  $\vec{a} = 2\vec{i} - \vec{j} + 2\vec{k}$ , find a unit vector in the direction opposite to  $\vec{a}$ .

- (4) 3. Find  $\vec{a} \cdot \vec{b}$  if  $|\vec{a}| = 12$ ,  $|\vec{b}| = 15$  and the angle between  $\vec{a}$  and  $\vec{b}$  is  $\frac{\pi}{6}$  radians.

$\vec{a} \cdot \vec{b} =$

- (6) 4. Find the values of  $t$  for which the vectors  $\langle 3t, -t, -3 \rangle$  and  $\langle -1, t^2, -4t \rangle$  are orthogonal.

$t =$
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- (4) 5. A constant force  $\vec{F} = 3\vec{i} + 5\vec{j} + 10\vec{k}$  moves an object along the line segment from  $(1, 0, 2)$  to  $(5, 3, 8)$ . Find the work done if the distance is measured in meters and the force in newtons.

$W =$ <span style="float: right;">J</span>
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- (6) 6. Find the area of the parallelogram determined by the vectors  $\vec{a} = \vec{i} + 5\vec{j} + \vec{k}$  and  $\vec{b} = -2\vec{i} + \vec{j} + 3\vec{k}$ .

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- (8) 7. Find the center and radius of the sphere

$$x^2 + y^2 + z^2 + 2x - 10y = -1.$$

center: radius:
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- (10) 8. The region bounded by the curves  $x = y^2 + 1$ ,  $y = -1$ ,  $y = 1$ , and  $x = 0$  is rotated about the  $y$ -axis. Find the volume of the resulting solid.

- (10) 9. Find the volume of the following solid  $S$  : The base of  $S$  is a circular disk with radius  $r$ . Parallel cross-sections perpendicular to the base are squares.

- (8) 10. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the curves  $x = \sqrt{\sin y}$  with  $0 \leq y \leq \pi$ , and  $x = 0$ , about the line  $y = 4$ .

- (8) 11. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the curves  $y = x^2 + 1$ ,  $y = 9 - x^2$  about the line  $y = -1$ .

- (6) 12. The natural length of a spring is 1m and a force of 10N is required to hold the spring stretched to a total length of 2m. How much work is done in stretching the spring from its natural length to a length of 1.5m?

$W =$  J

- (16) 13. Evaluate the integrals.  
 (a)  $\int \tan^{-1} x \, dx$

(b)  $\int_1^2 \frac{\ln x}{x^2} \, dx$