MA 16600
EXAM 1 INSTRUCTIONS
VERSION 01
February 6, 2013

Your name ______________________ Your TA's name ______________________

Student ID # ________________ Section # and recitation time ___________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the spaces provided for each of the questions 1–12. Do all your work on the question sheets. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

8. There are 12 questions, each worth 8 points. The maximum possible score is

$8 \times 12 + 4$ (for taking the exam) $= 100$ points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets.

Questions

1. Find the radius of the circle which is the intersection of the sphere given by the equation $(x-1)^2 + (y+2)^2 + (z-4)^2 = 21$ and the plane defined by $y = 2$.

A. 5

B. $\sqrt{5}$

C. $\sqrt{21}$

D. 2

E. $\sqrt{2}$

2. Find the vector projection $\text{proj}_{\vec{a}}\vec{b}$ of $\vec{b} = \langle 3, -1, 2\rangle$ onto $\vec{a} = \langle 2, 1, 0\rangle$.

A. $\langle 2\sqrt{5}, \sqrt{5}, 0\rangle$

B. $\langle 2, 1, 0\rangle$

C. 1

D. $\sqrt{5}$

E. $\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\rangle$

3. Find the unit vector $\vec{u}$ with the following properties:

(a) The vector $\vec{u}$ is perpendicular to the plane containing the three points $P(0, 1, 2)$, $Q(2, 2, 1)$, $R(1, 3, 1)$.

(b) The $\vec{k}$ component of $\vec{u}$ is negative.

A. $\langle 1, 1, 3\rangle$

B. $\frac{1}{\sqrt{11}}\langle 1, 1, 3\rangle$

C. $\langle -1, -1, -3\rangle$

D. $\frac{1}{\sqrt{11}}\langle -1, -1, -3\rangle$

E. $\frac{1}{\sqrt{11}}\langle -1, 1, -3\rangle$

4. Find the area of the region enclosed by the curves

$$y = x^3, y = |x|$$

over the interval $[-1, 1]$.

A. $\frac{1}{2}$

B. $\frac{3}{4}$

C. 1

D. $\frac{7}{4}$

E. 2

5. Determine

① the value of s so that the vector $\vec{v}_s = \langle 3s-1, 2, s\rangle$ is perpendicular to $\vec{w} = \langle 2, -1, -2\rangle$,

② the value of t so that the vector $\vec{v}_t = \langle 3t-1, 2, t\rangle$ is parallel to $\vec{w} = \langle 2, -1, -2\rangle$.

A. $s = 2, t = -2$

B. $s = 2, t = 1$

C. $s = 1, t = -2$

D. $s = 1$, no such value for t

E. no such value for s, no such value for t

6. A tank has a shape of an inverted circular cone with height 12 m and base radius 5 m. It is filled with water to a height of 7 m. Find the formula for the work required to empty the tank by pumping all the water to the top of the tank. (The density of water is 1000kg/m^3, and the acceleration due to gravity is 9.8m/s^2. The variable x represents the depth from the bottom, i.e., from the vertex of the cone.)

A. $\int_0^7 9.8 \times 1000 \times x\pi \left\{\frac{5}{12}(12-x)\right\}^2 dx$

B. $\int_0^7 9.8 \times 1000 \times x\pi \left\{\frac{5}{7}(12-x)\right\}^2 dx$

C. $\int_3^{10} 9.8 \times 1000 \times x\pi \left\{\frac{5}{7}(12-x)\right\}^2 dx$

D. $\int_7^0 9.8 \times 1000 \times (12-x)\pi \left(\frac{5}{12}x\right)^2 dx$

E. $\int_0^7 9.8 \times 1000 \times (12-x)\pi \left(\frac{5}{12}x\right)^2 dx$

7. Compute the volume of the northern cap of a sphere with radius 2 and height 1 (see the picture below).

A. $\frac{4\pi}{3}$

B. $\frac{5\pi}{3}$

C. $\frac{7\pi}{3}$

D. $\frac{8\pi}{3}$

E. $\frac{16\pi}{3}$

8. Find the formula for the volume of the solid obtained by rotating the region enclosed by the curves

$$y = 3 + 2x - x^2, y = -x + 3$$

about $x = 5$ using the method of cylindrical shells.

A. $\int_0^5 2\pi x\{(3 + 2x - x^2) - (-x + 3)\}dx$

B. $\int_0^3 2\pi x\{(3 + 2x - x^2) - (-x + 3)\}dx$

C. $\int_0^3 2\pi(5 - x)\{(-x + 3) - (3 + 2x - x^2)\}dx$

D. $\int_0^3 2\pi(5 - x)\{(3 + 2x - x^2) - (-x + 3)\}dx$

E. $\int_0^5 2\pi(5 - x)\{(3 + 2x - x^2) - (-x + 3)\}dx$

9. Find the volume of the described solid S:

The base of S is the triangular region with vertices $(0, 0), (0, 2)$ and $(1, 0)$.

Cross sections perpendicular to the x-axis are squares.

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{4}{3}$

D. $\frac{8}{3}$

E. $\frac{10}{3}$

10. Find the positive number b such that the average value of $f(x) = 3x^2 - 2x - 5$ on the interval $[0, b]$ is equal to 1.

A. 1

B. 2

C. 3

D. 4

E. 5

11. Calculate $\int_0^1 \tan^{-1} x \; dx$.

A. $\frac{\pi}{4} - \frac{\ln 2}{2}$

B. $\frac{\pi}{4} + \frac{\ln 2}{2}$

C. $\frac{\pi}{4}$

D. $\frac{\ln 2}{2}$

E. ∞

12. Integration by parts gives us

$$\int e^x \cos x \; dx = e^x \sin x - \int e^x \sin x \; dx.$$

and

$$\int e^x \sin x \; dx = -e^x \cos x + \int e^x \cos x \; dx.$$

From this, we derive a formula for the indefinite integral $\int e^x \cos x \; dx$ as

A. $e^x(\sin x - \cos x) + C$

B. $\frac{1}{2}e^x(\sin x - \cos x) + C$

C. $\frac{1}{2}e^x(\sin x + \cos x) + C$

D. $\frac{1}{2}e^x(\cos x - \sin x) + C$

E. We observe that the method of integration by parts becomes circular in this case, and that we can never derive a concrete formula.