MA 16600
EXAM 1 INSTRUCTIONS
VERSION 01
February 12, 2014

Your name ______________________ Your TA's name ______________________

Student ID # ________________ Section # and recitation time ____________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your question booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the spaces provided for each of the questions 1–12. Do all your work on the question sheets. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

8. There are 12 questions, each worth 8 points. The maximum possible score is

$8 \times 12 + 4$ (for taking the exam) $= 100$ points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheets and the exam booklets.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ________________________________

STUDENT SIGNATURE: ________________________________

Questions

1. Determine the value of a so that the sphere defined by the equation

$$x^2 + y^2 + z^2 - 4x + 2y - 10z = a$$

has radius $r = 4$.

A. 30
B. 15
C. 9
D. -14
E. -28

2. We have the following two vectors

$$\begin{cases} \vec{a} &= \langle 2, 0, 3 \rangle \\ \vec{b} &= \langle 4, -1, 3 \rangle \end{cases}$$

Find the vector projection $\mathbf{Proj}_{\vec{a}}\vec{b}$ of $\vec{b}$ onto $\vec{a}$.

A. $\langle 4\sqrt{5}, 0, 12\sqrt{5} \rangle$

B. $\langle 2, -1, \frac{2}{3} \rangle$

C. $\langle \frac{34}{13}, 0, \frac{51}{13} \rangle$

D. $\langle \frac{4}{13}, -\frac{1}{13}, \frac{3}{13} \rangle$

E. $\langle \frac{34}{\sqrt{13}}, 0, \frac{51}{\sqrt{13}} \rangle$

3. Find the area of the triangle with vertices $P(1, -1, 0), Q(3, -2, 2)$ and $R(2, -2, 1)$.

A. 6

B. 3

C. $\sqrt{2}$

D. $2\sqrt{2}$

E. $\frac{\sqrt{2}}{2}$

4. Let

$$\begin{cases} \vec{a} &= \langle 1, -4, 2 \rangle, \\ \vec{b} &= \langle -1, 3, -1 \rangle. \end{cases}$$

Find a unit vector $\vec{u}$ which satisfies the following two conditions:

(a) $\vec{u}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, and

(b) the $\vec{k}$ component of $\vec{u}$ is negative.

A. $\langle 1, 1, -2 \rangle$

B. $\frac{1}{\sqrt{11}}\langle 1, 1, 3 \rangle$

C. $\frac{1}{\sqrt{6}}\langle -2, -1, -1 \rangle$

D. $\frac{1}{\sqrt{11}}\langle -3, -1, -1 \rangle$

E. $\frac{1}{\sqrt{3}}\langle -1, 1, 1 \rangle$

5. Find the area of the region bounded by the curves

$$\begin{cases} y &= \sin x, \\ y &= \cos x, \end{cases}$$

and the lines

$$\begin{cases} x &= -\pi/2 \\ x &= \pi/2. \end{cases}$$

A. $\frac{\sqrt{2}}{2}$

B. $\sqrt{2}$

C. $2\sqrt{2}$

D. $2\sqrt{2} - 2$

E. $\sqrt{2} - 1$

6. Let R be the region enclosed by the curves $y = 8x$ and $y = 2x^2 + 6$.

Which of the following represents the volume V, computed by using the Washer Method, of the solid generated by revolving the region R about the axis $y = -1$?

A. $\int_1^3 2\pi x\,[8x - (2x^2 + 6)]\,dx$

B. $\int_1^3 2\pi x\,[(2x^2 + 6) - 8x]\,dx$

C. $\int_1^3 \pi\,[(8x - 1)^2 - (2x^2 + 5)^2]\,dx$

D. $\int_1^3 \pi\,[(8x)^2 - (2x^2 + 6)^2]\,dx$

E. $\int_1^3 \pi\,[(8x + 1)^2 - (2x^2 + 7)^2]\,dx$

7. Let R be the region enclosed by the curve $y = \sqrt{x-1}$, $y = 0$, and $x = 2$.

Which of the following represents the volume V, computed by using the Cylindrical Shell Method, of the solid generated by revolving the region R about the y-axis.

A. $\int_1^2 \pi(\sqrt{x-1})^2 dx = \int_1^2 \pi(x-1)dx$

B. $\int_1^2 2\pi x\sqrt{x-1}dx$

C. $\int_{-2}^2 2\pi x\sqrt{x-1}dx$

D. $\int_0^1 2\pi y\,[2-(y^2+1)]\,dy$

E. $\int_0^1 \pi\,[4-(x^2+1)^2]\,dy$

8. A tank has the shape obtained by rotating the curve $y = x^4, 0 \leq x \leq 3$ about the y-axis. The tank is full of liquid, which weighs 1 lb/ft^3. Set up an integral for the work required to empty the tank by pumping all the liquid from the top of the tank.

Note: The units in the x-axis and in the y-axis are measured to be 1 ft in length.

A. $\int_0^3 2\pi x(81 - x^4)dx$ ft-lb

B. $\int_0^{81} \pi y^2(81 - y)dy$ ft-lb

C. $\int_0^3 \pi x^2(81 - x^4)dx$ ft-lb

D. $\int_0^{81} \pi\sqrt{y}(81 - y)dy$ ft-lb

E. $\int_0^3 \pi x^2(81 - y)dx$ ft-lb

9. Find the volume of the solid S described below:

(a) The base of S is the circle with radius 1 on the x-y plane.

(b) Cross sections perpendicular to the x-axis are equilateral triangles.

A. $\frac{\sqrt{3}}{3}$

B. $\frac{2\sqrt{3}}{3}$

C. $\frac{4\sqrt{3}}{3}$

D. $\frac{1}{3}$

E. $\frac{2}{3}$

10. Suppose that f is a continuous function on the closed interval $[1, 4]$ such that $\int_1^4 f(x)dx = 18$. Which of the following statements are true ?

(I) Set $F(x) = \int_1^x f(t)dt$. (Then by Fundamental Theorem of Calculus, we have $F'(x) = f(x)$). Via Mean Value Theorem applied to the function F, we conclude that there is a value $c \in (1, 4)$ such that $f(c) = F'(c) = \frac{F(4)-F(1)}{4-1} = \frac{18-0}{4-1} = 6$.

(II) The given information implies that, if $f(1) \geq 6$, then $f(4) \leq 6$, and that, if $f(1) \leq 6$, then $f(4) \geq 6$. Therefore, by Intermediate Value Theorem, we conclude that there is a value $c \in (1, 4)$ such that $f(c) = 6$.

(III) We can not determine, from the given information only, whether there is a value $c \in (1, 4)$ such that $f(c) = \frac{18}{4-1} = 6$, the average value.

A. (I) ONLY.

B. (II) ONLY.

C. (III) ONLY.

D. (I) and (II) ONLY.

E. None of the above.

11. Evaluate $\int_2^{\ln 5} xe^x dx$.

A. $5\ln 5 + 3e^2$

B. $45 - e^2$

C. $4\ln 5 - e^2$

D. $45 + e^2$

E. $5\ln 5 - 5 - e^2$

12. Evaluate

$$\int_0^{(\pi/2)^2} \cos(\sqrt{x})dx.$$

HINT: Set $\sqrt{x} = u$. Then $x = u^2$, and hence $dx = 2udu$.

A. $\frac{\pi}{2} + 1$

B. $\pi - 2$

C. $\frac{2\pi}{3} - \frac{1}{2}$

D. $\frac{\pi}{3} + 1$

E. $2\pi + 2$