MA 16600
EXAM 1 INSTRUCTIONS
VERSION 01
February 17, 2020

Your name _____ Your TA's name _____

Student ID # _____ Section # and recitation time _____

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit **SECTION NUMBER**.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–12. While mark all your work on the scantron sheet, you should show your work on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. There are 12 questions, each worth 8 points. The maximum possible score is
$8 \times 12 + 4$ (for taking the exam) $= 100$ points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

Questions

1. Write inequalities to describe the upper hemisphere of the solid sphere (i.e., ball) of radius 5 centered at $(-1, 3, -5)$.

(Note: We choose the z-axis to be the vertical one, while the xy-plane is horizontal. The word "upper" is with respect to the vertical z-axis.)

A. $(x+1)^2 + (y-3)^2 + (z+5)^2 \geq 5^2, \quad z \leq -5$

B. $(x-1)^2 + (y-3)^2 + (z-5)^2 \geq 5^2, \quad z \leq -5$

C. $(x+1)^2 + (y-3)^2 + (z+5)^2 \leq 5^2, \quad z \geq -5$

D. $(x-1)^2 + (y-3)^2 + (z-5)^2 \leq 5^2, \quad z \leq -5$

E. $(x-1)^2 + (y-3)^2 + (z-5)^2 \leq 5^2, \quad z \geq -5$

2. Find a vector that has the same direction as the vector $\overrightarrow{PQ}$ but has length 3, where P is the point $(-1, 3, -2)$ and Q is the point $(1, -2, 3)$.

A. $\frac{1}{6}\langle 2, -5, 5\rangle$

B. $\frac{1}{6}\langle -2, 5, -5\rangle$

C. $\frac{1}{\sqrt{6}}\langle -2, 5, -5\rangle$

D. $\frac{1}{\sqrt{6}}\langle 2, -5, 5\rangle$

E. $\frac{1}{3}\langle -2, 5, -5\rangle$

3. Find the $\text{Proj}_{\mathbf{v}}\mathbf{u}$, where $\mathbf{u} = \langle 4, 0, 5\rangle$ and $\mathbf{v} = \langle 4, 3, 0\rangle$.

A. $\frac{16}{25}\mathbf{v}$

B. $\frac{16}{25}\mathbf{u}$

C. $\frac{16}{5}\mathbf{v}$

D. $\frac{16}{5}\mathbf{u}$

E. $\frac{9}{25}\mathbf{v}$

4. Find the value of k so that the following 4 points are coplanar (i.e., they are on the same plane):

$$\begin{aligned} P &= (1,1,0) \\ Q &= (1,2,k) \\ R &= (3,2,2) \\ S &= (5,0,7) \end{aligned}$$

A. 1

B. -1

C. $\dfrac{1}{3}$

D. $-\dfrac{1}{3}$

E. No such value exists for k.

5. Find the angle θ between the following two tangent lines:
one is to the curve $y = 3x^3$ at point $(1, 3)$ and
the other to the curve $y = 3x^{\frac{1}{3}}$ at point $(1, 3)$.

A. $\dfrac{\pi}{3}$

B. $\dfrac{\pi}{6}$

C. $\arccos\left(\dfrac{3}{\sqrt{41}}\right)$

D. $\arccos\left(\dfrac{5}{\sqrt{35}}\right)$

E. $\arccos\left(\dfrac{5}{\sqrt{41}}\right)$

6. Let $\mathbf{u} = \langle 1, -1, 2 \rangle$.

Find a vector $\mathbf{v}$ such that $\mathbf{v}$ is perpendicular to $\mathbf{u}$ and $\mathbf{u} \times \mathbf{v} = \langle -7, 1, 4 \rangle$.

A. $\langle 1, 1, 0 \rangle$

B. $\langle 1, 3, 1 \rangle$

C. $\langle 3, 3, 0 \rangle$

D. $\langle 2, 6, 2 \rangle$

E. $\langle 3, 9, 3 \rangle$

7. Find the area of the region bounded by the curves $y = x^3$ and $y = x - 6$ and the x-axis.

A. $8\sqrt{6}$

B. $12\sqrt{2}$

C. $6\sqrt{3}$

D. 14

E. 28

8. Choose the right formula to compute the volume of the solid obtained from rotating the region bounded by $y = x^2, x = 2$ and $y = 0$ about $x = 3$ using

(i) Washer method,

(ii) Shell method.

A. (i) $\displaystyle\int_0^4 \pi(\sqrt{y})^2 dy$, (ii) $\displaystyle\int_0^2 2\pi x \cdot x^2 dx$

B. (i) $\displaystyle\int_0^2 2\pi x \cdot x^2 dx$, (ii) $\displaystyle\int_0^4 \pi\left\{(\sqrt{y})^2 - 2^2\right\} dy$

C. (i) $\displaystyle\int_0^4 \pi\left\{(3-\sqrt{y})^2 - 1^2\right\} dy$, (ii) $\displaystyle\int_0^2 2\pi(3-x)x^2 dx$

D. (i) $\displaystyle\int_0^4 \pi\left\{(3+\sqrt{y})^2 - 1^2\right\} dy$, (ii) $\displaystyle\int_0^2 2\pi(x-3)x^2 dx$

E. (i) $\displaystyle\int_0^3 2\pi(3-x)x^2 dx$, (ii) $\displaystyle\int_0^4 \pi(3-\sqrt{y})^2 dy$

9. The base of a solid is the region bounded by the parabolas $y = x^2$ and $y = 2 - x^2$.

Find the volume of the solid if the cross-sections perpendicular to the base and parallel to the y-axis are equilateral triangles with one side lying along the base.

A. $\frac{1}{3}\sqrt{3}$

B. $\frac{16}{15}\sqrt{3}$

C. $\frac{8}{15}\sqrt{3}$

D. $\frac{3}{5}\sqrt{3}$

E. $\frac{6}{5}\sqrt{3}$

HINT: The area of an equilateral triangle with base length B is give by the formula $\frac{\sqrt{3}}{4}B^2$.

WARNING: The region above lies not only over the 1st quadrant but also over the 2nd quadrant.

10. Find the length of the curve

$y = f(x) = \dfrac{e^x + e^{-x}}{2}$ on the interval $[0, \ln 3]$.

A. $\sqrt{\dfrac{19}{3}}$

B. $\ln 3 + \dfrac{4}{3}$

C. $\ln 3 + \dfrac{1}{3}$

D. $\dfrac{4}{3}$

E. $\dfrac{8}{3}$

11. Find the area of the northern cap of the sphere of radius 1 when the cap has height 1/2.

A. π

B. π^2

C. $\dfrac{4\pi}{3}$

D. $\dfrac{2\pi}{3}$

E. $\dfrac{\pi}{4}$

12. An aquarium has the shape of a box with all sides perpendicular to each other. Its sides measure 2 m long, 5 m wide, and 1 m deep is full of water. Find the work in Joules needed to pump all the water out of the aquarium.

Note:

- Use the fact that the density of water is 1000 kg/m^3.
- Use g m/s^2 for the accceleration due to the gravity.

A. $10^4 g$ J

B. $10^3 g$ J

C. $5000g$ J

D. $2500g$ J

E. $500g$ J