MA 16600
EXAM 2 INSTRUCTIONS
VERSION 01
March 10, 2020

Your name ____________________ Your TA's name ____________________

Student ID # ________________ Section # and recitation time ____________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit **SECTION NUMBER**.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–12. While **mark all your work on the scantron sheet**, you should **show your work on the exam booklet**. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. There are 12 questions, each worth 8 points. The maximum possible score is

$8 \times 12 + 4$ (for taking the exam) $= 100$ points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

Questions

1. Evaluate the integral

$$\int_0^{\frac{\pi}{2}} x\sin(2x)dx.$$

A. $\dfrac{\pi}{2}$

B. $\dfrac{\pi}{3}$

C. $\dfrac{\pi}{4}$

D. $-\dfrac{\pi}{4}$

E. $-\dfrac{\pi}{3}$

2. Three students try to compute

$$\int \tan(x)\ dx = \int \frac{\sin(x)}{\cos(x)}\ dx$$

using the substitution $u = \sin x$.

Mark: After the substitution, the integration becomes

$\displaystyle\int \frac{u}{1-u^2}\ du = \frac{1}{2}\int \left(\frac{1}{1-u} - \frac{1}{1+u}\right)\ du.$

Therefore, the final answer is

$\displaystyle\frac{1}{2}\left(-\ln|1-\sin(x)| - \ln|1+\sin(x)|\right) + C.$

Sasha: If we use one more substitution $v = 1-u^2$, the integration becomes

$\displaystyle\int \frac{u}{1-u^2}\ du = -\frac{1}{2}\int \frac{dv}{v}.$

Therefore, the final answer is $-\dfrac{1}{2}\ln|1-\sin^2(x)| + C$.

Rachel: If we want to use this substitution, then we should have $du = \cos(x)\ dx$. But in the formula $\displaystyle\int \frac{\sin(x)}{\cos(x)}\ dx$ we have only $\dfrac{1}{\cos(x)}\ dx$ but not $\cos(x)\ dx$. So we cannot use the substitution $u = \sin(x)$ to compute the integration.

Choose the correct statement about their claims from the following.

A. ONLY Mark is right.

B. ONLY Sasha is right.

C. ONLY Rachel is right.

D. BOTH Mark and Sasha are right, and Rachel is wrong.

E. The strategies by Mark and Sasha are right, but their answers are wrong. They are different from the well-known formula
$\displaystyle\int \tan(x)\ dx = \ln|\sec(x)| + C.$

3. Compute $\int \sin^5(x)\cos^2(x)dx$.

A. $-\frac{1}{3}\cos^2(x) + \frac{2}{5}\cos^4(x) - \frac{1}{7}\cos^6(x) + C$

B. $-\frac{1}{3}\cos^3(x) + \frac{2}{5}\cos^4(x) - \frac{1}{7}\cos^6(x) + C$

C. $-\frac{1}{3}\cos^3(x) + \frac{2}{5}\cos^5(x) - \frac{1}{7}\cos^6(x) + C$

D. $-\frac{1}{3}\cos^3(x) + \frac{2}{5}\cos^5(x) - \frac{1}{7}\cos^7(x) + C$

E. $\frac{1}{3}\cos^2(x) - \frac{2}{5}\cos^5(x) + \frac{1}{7}\cos^7(x) + C$

4. Compute $\int \sin^2(x)\cos^2(x)dx$.

A. $\frac{1}{4}x - \frac{1}{16}\cos(4x) + C$

B. $\frac{1}{8}x + \frac{1}{32}\sin(2x) + C$

C. $\frac{1}{4}x - \frac{1}{16}\sin(4x) + C$

D. $\frac{1}{8}x - \frac{1}{32}\cos(2x) + C$

E. $\frac{1}{8}x - \frac{1}{32}\sin(4x) + C$

5. Compute the integral $\int \tan^3(x)\sec^5(x)dx$.

A. $\frac{1}{7}\sec^7(x) + \frac{1}{5}\sec^5(x) + C$
B. $\frac{1}{6}\sec^5(x) - \frac{1}{4}\sec^3(x) + C$
C. $\frac{1}{7}\sec^7(x) - \frac{1}{5}\sec^5(x) + C$
D. $\frac{1}{7}\sec^7(x) - \frac{1}{5}\sec^3(x) + C$
E. $\frac{1}{6}\sec^6(x) - \frac{1}{4}\sec^4(x) + C$

6. We try to compute

$$\int \tan^2(x)\sec(x)\ dx$$

using integration by parts, setting

$$\begin{cases} u = \tan(x), & v = \sec(x) \\ du = \sec^2(x)\ dx, & dv = \sec(x)\tan(x)\ dx. \end{cases}$$

Then we have

$$\begin{aligned} \int \tan^2(x)\sec(x)dx &= \int udv = uv - \int vdu \\ &= \tan(x)\sec(x) - \int \sec(x)\sec^2(x)\ dx \\ &= \tan(x)\sec(x) - \int \sec(x)\{1+\tan^2(x)\}\ dx \\ &= \tan(x)\sec(x) - \int \sec(x)\ dx - \int \tan^2(x)\sec(x)dx. \end{aligned}$$

Choose the correct statement for the above trial.

A. Since we end up seeing the same formula at the end as we started with, this trial fails to compute the integral.

B. The only substitution we can use is $u = \tan(x)$ or $u = \sec(x)$. For the given integral, neither of them works. The above trial confirms that this integral cannot be computed by any method.

C. This computation gives the formula
$$\int \tan^2(x)\sec(x)\ dx = \tan(x)\sec(x) - \ln|\sec(x)+\tan(x)| + C.$$

D. This computation gives the formula
$$\int \tan^2(x)\sec(x)\ dx = \frac{1}{2}\{\tan(x)\sec(x) - \ln|\sec(x)+\tan(x)|\} + C.$$

E. The above trial fails. We have to use the foumula
$$\int \tan^2(x)\sec(x)\ dx = \int \{\sec^2(x)-1\}\sec(x)\ dx.$$
Then ONLY after we compute $\int \sec^3(x)\ dx$ and $\int \sec(x)\ dx$, we can derive the formula.

7. Evaluate the integral

$$\int_{\frac{1}{3}}^{\frac{\sqrt{3}}{3}} \frac{dx}{x^2\sqrt{9x^2+1}}.$$

A. $3\sqrt{2} - 2\sqrt{3}$

B. $\sqrt{2} - \dfrac{2\sqrt{3}}{3}$

C. $\dfrac{\sqrt{3}}{2} - \frac{1}{2}$

D. $\sqrt{3} - 1$

E. $\sqrt{2} - 1$

8. Write out the form of the partial decomposition of the function

$$\frac{1}{(x+2)(x^2-4)(x^2+x+1)}.$$

A. $\dfrac{A}{x+2} + \dfrac{Bx+C}{x^2-4} + \dfrac{Dx+E}{x^2+x+1}$

B. $\dfrac{A}{x+2} + \dfrac{B}{x^2-4} + \dfrac{Cx+D}{x^2+x+1}$

C. $\dfrac{A}{x-2} + \dfrac{C}{(x+2)^2} + \dfrac{Dx+E}{x^2+x+1}$

D. $\dfrac{A}{x-2} + \dfrac{Bx+C}{(x+2)^2} + \dfrac{Dx+E}{x^2+x+1}$

E. $\dfrac{A}{x-2} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2} + \dfrac{Dx+E}{x^2+x+1}$

Note: The letters A, B, C, D, E in the partial fractions above represent some appropriate constants.

9. Compute the integral

$$\int \frac{x^2 + x + 2}{x^2 + 2x + 2}\, dx.$$

A. $x - \ln(x^2 + 2x + 2) + C$

B. $x - \ln(x^2 + 2x + 2) + \tan^{-1}(x + 1) + C$

C. $x - \frac{1}{2}\ln(x^2 + 2x + 2) + \tan^{-1}(x + 1) + C$

D. $x + \ln(x^2 + 2x + 2) - \frac{1}{2}\tan^{-1}(x + 1) + C$

E. $\frac{1}{2}x\ln(x^2 + 2x + 2) + 2\tan^{-1}(x + 1) + C$

10. Evaluate the improper integral $\int_{-\infty}^{0} xe^x dx$.

A. $-\frac{1}{4}$

B. -1

C. $-\frac{1}{2}$

D. $-\frac{1}{3}$

E. The integral is divergent.

11. Compute the integral $\int \frac{x}{\sqrt{5+4x-x^2}}dx$.

A. $2\cos^{-1}\left(\frac{x-2}{3}\right) - \sqrt{5+4x-x^2} + C$

B. $2\sin^{-1}\left(\frac{x-2}{3}\right) - \sqrt{5+4x-x^2} + C$

C. $2\sin^{-1}\left(\frac{x-2}{3}\right) + \sqrt{5+4x-x^2} + C$

D. $2\cos^{-1}\left(\frac{\sqrt{5+4x-x^2}}{3}\right) - \sqrt{5+4x-x^2} + C$

E. $2\cos^{-1}\left(\frac{x-2}{3}\right) - \frac{\sqrt{5+4x-x^2}}{2} + C$

12. Compute the following limit

$$\lim_{n\to\infty} \frac{(2n)!n^2}{(2n+2)!} \cos\left(\frac{1}{n}\right).$$

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. 1

E. The limit does not exist.