

NAME \_\_\_\_\_

10-DIGIT PUID \_\_\_\_\_

RECITATION INSTRUCTOR \_\_\_\_\_

RECITATION TIME \_\_\_\_\_

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TOTAL	/100

## DIRECTIONS

- Write your name, 10-digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, and 4.
- The test has four (4) pages, including this one.
- Write your answers in the boxes provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses in the left hand margin.
- No books, notes, calculators, or any electronic devices may be used on this test.

(12) 1. Circle T if True or F if False. You do not need to show work.

(a) If  $\lim_{n \rightarrow \infty} na_n = 5$ , then  $\sum_{n=1}^{\infty} a_n$  converges.

T F

(b) There is a convergent alternating series  $\sum_{n=1}^{\infty} (-1)^n b_n$ , with  $b_n > 0$ , and such that  $\lim_{n \rightarrow \infty} b_n \neq 0$ .

T F

(c) If  $\sum_{n=1}^{\infty} a_n$  diverges, then  $\sum_{n=1}^{\infty} |a_n|$  diverges.

T F

(12) 2. Determine whether each of the following series is convergent or divergent. You do not need to show work.

(a)  $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$

(b)  $\sum_{n=1}^{\infty} \frac{2 + \sin n}{n^2}$

(c)  $\sum_{n=1}^{\infty} (2^{1/n} - 1)^n$

(5) 3.  $\sum_{n=1}^{\infty} \frac{3^n}{4^n} =$

- (27) 4. Determine whether each series is convergent or divergent. You must state the conditions of the test you are using and verify them if they are not obvious. Write your conclusion in the small box.

(a)  $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \dots (2n - 1)}{5^n n!}$

Show all necessary work here:

By the \_\_\_\_\_ test, the series is \_\_\_\_\_

(b)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n5^n}$

Show all necessary work here:

By the \_\_\_\_\_ test, the series is \_\_\_\_\_

(c)  $\sum_{n=1}^{\infty} \frac{n+2}{\sqrt{n^3+1}}$

Show all necessary work here:

By the \_\_\_\_\_ test, the series is \_\_\_\_\_

(10) 5. The series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n(10)^n}$  satisfies the conditions of the alternating series test.

(a) Write out the first five terms of the series.

(b) Use the alternating series estimation theorem to find the number of terms that we need to add in order to estimate the sum of the series with error  $< 0.0001$ .

\_\_\_\_\_ terms

(5) 6. Find the Taylor series of the function  $f(x) = e^x$  centered at  $a = -1$ .

$e^x = \sum$

(14) 7. For the power series  $\sum_{n=1}^{\infty} \frac{n}{5^n} (x+1)^n$ , find the following, showing all work.

(a) The radius of convergence  $R$ .

$R =$
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(b) The interval of convergence. (Don't forget to check the end points).

Interval of convergence
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(15) 8. For each function  $f$  find its Maclaurin series and radius of convergence. You may use known series to get your answer.

(a)  $f(x) = \frac{1}{1-3x}$

$\frac{1}{1-3x} = \sum$	, $R =$
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(b)  $f(x) = \frac{1}{(1+x)^2}$ . (Hint:  $\frac{d}{dx} \frac{1}{1+x} = -\frac{1}{(1+x)^2}$ )

$\frac{1}{(1+x)^2} = \sum$	, $R =$
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(c)  $f(x) = e^{-x^2}$

$e^{-x^2} = \sum$	, $R =$
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