MA166 — EXAM III — SPRING 2019 — APRIL 9, 2019
TEST NUMBER 11

INSTRUCTIONS:

1. **Do not open the exam booklet until you are instructed to do so.**
2. **Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
3. **MARK YOUR TEST NUMBER ON YOUR SCANTRON**
4. Once you are allowed to open the exam, make sure you have a complete test. There are 7 different test pages (including this cover page).
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers on this test booklet.
6. The exam has 12 problems and each one is worth 8 points and everyone gets four points. The maximum possible score is 100 points. No partial credit.
7. Do not leave the exam room during the first 20 minutes of the exam.
8. If you do not finish your exam in the first 40 minutes, you must wait until the end of the exam period to leave the room.
9. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

DON'T BE A CHEATER:

1. Do not give, seek or obtain any kind of help from anyone to answer questions on this exam. If you have doubts, consult only your instructor.
2. Do not look at the exam or scantron of another student.
3. Do not allow other students to look at your exam or your scantron.
4. You may not compare answers with anyone else or consult another student until after you have finished your exam, given it to your instructor and left the room.
5. Do not consult notes or books.
6. **Do not handle** phones or cameras, calculators or any electronic device until after you have finished your exam, given it to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

STUDENT ID NUMBER: ______________________________

SECTION NUMBER AND RECITATION INSTRUCTOR: ______________________________

1. Compute the series

$$\sum_{n=1}^{\infty} \frac{2^{n-1} - 3^n}{5^n}.$$

A. $\frac{1}{3}$

B. $-\frac{3}{2}$

C. $-\frac{7}{6}$

D. $\frac{5}{6}$

E. The series is divergent.

2. Test the following series for convergence:

(i) $\sum_{n=1}^{\infty} \frac{1}{n+1}$

(ii) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

(iii) $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$

(iv) $\sum_{n=1}^{\infty} \frac{2^n}{3^n - n}$

(v) $\sum_{n=1}^{\infty} \frac{1-\sqrt{n}}{n^3}$

A. (ii) is convergent; (i), (iii), (iv) and (v) are divergent.

B. (ii) and (v) are convergent; (i), (iii) and (iv) are divergent.

C. (iv) and (v) are convergent; (i), (ii) and (iii) are divergent.

D. (iv) is convergent; (i), (ii), (iii) and (v) are divergent.

E. (ii), (iv) and (v) are convergent; (i) and (iii) are divergent.

3. Test the following series for convergence:

(i) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\sqrt{n}}$

(ii) $\sum_{n=2}^{\infty}\frac{1}{n\ln(n)}$

(iii) $\sum_{n=2}^{\infty}(-1)^{n-1}\frac{1}{n\ln(n)}$

(iv) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n\sqrt{n}}$

A. (i) and (iii) are convergent; (ii) and (iii) are divergent.

B. (i), (iii) and (iv) are convergent; (ii) is divergent.

C. (i) is convergent; (ii), (iii) and (iv) are divergent.

D. (iii) and (iv) are convergent; (i) and (ii) are divergent.

E. (iii) is convergent; (i), (ii) and (iv) are divergent.

4. Which of the following statements are true?

(i) If $a_n \leq \frac{1}{\ln n}$ for $n \geq 2$, then the series $\sum_{n=2}^{\infty} a_n$ is convergent.

(ii) The series $\sum_{n=1}^{\infty}(-1)^n\sqrt{\frac{n+2}{3n-1}}$ is convergent.

(iii) If $0 \leq b_n \leq \frac{1}{\ln n}$ and $b_n \geq b_{n+1}$ for $n \geq 2$, then the series $\sum_{n=2}^{\infty}(-1)^n b_n$ is convergent.

A. None

B. (i) only.

C. (ii) only.

D. (i) and (ii).

E. (iii) only.

5. Which of the following statements are true?

(i) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty}(-1)^n a_n$ converges.

(ii) If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty}(-1)^n a_n$ converges.

(iii) If $\lim_{n\to\infty} n^2|a_n| = 1$, then $\sum_{n=1}^{\infty}(-1)^n a_n$ absolutely converges.

A. (ii) and (iii) only.

B. (iii) only.

C. (ii) only.

D. (i) and (ii) only.

E. (i) and (iii) only.

6. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent:

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2+1}$

(ii) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{2n+1}$

A. (i) is conditionally convergent; (ii) is divergent.

B. (i) and (ii) are conditionally convergent.

C. (i) and (ii) are absolutely convergent.

D. (i) is absolutely convergent; (ii) is conditionally convergent.

E. (i) is conditionally convergent; (ii) is absolutely convergent.

7. Use the root test to decide which of the following statements are true

I) $\sum_{n=1}^{\infty}(-1)^{n-1}\left(\dfrac{2n^3+8n+1}{n^3+3n^2+7n}\right)^n$ diverges

II) $\sum_{n=1}^{\infty}\left(a+\dfrac{1}{n}\right)^n$ converges for $|a|<1$

III) If a sequence a_n is such that $-n-1 \le \ln|a_n| \le -n$ then $\sum_{n=1}^{\infty} a_n$ converges absolutely.

A. I, II and III are true

B. I, II and III are false

C. only I and II are true

D. only II and III are true

E. only I and III are true

8. Find the interval of convergence of the series $\sum_{n=2}^{\infty}\dfrac{x^n}{n\ln n}$

A. $[-1,1]$

B. $[-1,1)$

C. $(-1,1]$

D. $(-1,1)$

E. $[0,1]$

9. Find the radius of convergence R of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{2^n + n^2 + 6n}$

A. $R = 5$

B. $R = 4$

C. $R = 3$

D. $R = 2$

E. $R = 1$

10. The power series representation of the function $\dfrac{x}{1+x^2}$ is given by

A. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\, x^{2n+1}$ for $|x| < 1$

B. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\, x^{2n}$ for $|x| < 1$

C. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\, x^{2n+3}$ for $|x| < 1$

D. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n}\, x^{2n-1}$ for $|x| < 1$

E. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\, x^{2n-1}$ for $|x| < 1$

11. Given that $\arctan(x^2) = \int_0^x \frac{2t}{1+t^4}\,dt$, the Maclaurin series of the function $\arctan(x^2)$ for $|x| < 1$ is given by

A. $\arctan(x^2) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+4}}{n+2}$

B. $\arctan(x^2) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n+1}$

C. $\arctan(x^2) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{4n+2}$

D. $\arctan(x^2) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+8}}{3n+1}$

E. $\arctan(x^2) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{2n+1}$

12. We know that

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \text{ for } |x| < 1 \text{ and that } \ln x = \ln 2 + \ln(1 + \frac{x-2}{2}).$$

We then conclude that the Taylor series of $\ln x$ centered at 2 and for $|\frac{x-2}{2}| < 1$ is given by

A. $\ln x = \ln 2 + \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-2)^n}{n\,2^n}$

B. $\ln x = \ln 2 + \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-2)^n}{n^2}$

C. $\ln x = \ln 2 + \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-2)^n}{n}$

D. $\ln x = \ln 2 + \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-2)^n}{n\,2^{n+1}}$

E. $\ln x = \ln 2 + \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-2)^n}{n^{2n}}$

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TEST NUMBER 22

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SECTION NUMBER AND RECITATION INSTRUCTOR: ______________________________

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$$\sum_{n=1}^{\infty} \frac{2^n - 3^{n-1}}{5^n}.$$

A. $\frac{1}{2}$

B. $-\frac{3}{2}$

C. $\frac{5}{6}$

D. $\frac{1}{6}$

E. The series is divergent.

2. Test the following series for convergence:

(i) $\sum_{n=1}^{\infty} \frac{2}{n+2}$

(ii) $\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$

(iii) $\sum_{n=2}^{\infty} \frac{1}{n-\sqrt{n}}$

(iv) $\sum_{n=1}^{\infty} \frac{2^n}{3^n+n}$

(v) $\sum_{n=1}^{\infty} \frac{1+\sqrt{n}}{n^3}$

A. (ii), (iv) and (v) are convergent; (i) and (iii) are divergent.

B. (ii) and (v) are convergent; (i), (iii) and (iv) are divergent.

C. (iv) and (v) are convergent; (i), (ii) and (iii) are divergent.

D. (iv) is convergent; (i), (ii), (iii) and (v) are divergent.

E. (ii) is convergent; (i), (iii), (iv) and (v) are divergent.

3. Test the following series for convergence:

(i) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n}$

(ii) $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))}$

(iii) $\sum_{n=2}^{\infty}(-1)^n \frac{1}{n(\ln(n))}$

(iv) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n\sqrt{n}}$

A. (i) and (iii) are convergent; (ii) and (iii) are divergent.

B. (iii) and (iv) are convergent; (i) and (ii) are divergent.

C. (i) is convergent; (ii), (iii) and (iv) are divergent.

D. (i), (iii) and (iv) are convergent; (ii) is divergent.

E. (iii) is convergent; (i), (ii) and (iv) are divergent.

4. Which of the following statements are true?

(i) If $a_n \leq \frac{1}{n}$ for $n \geq 1$, then the series $\sum_{n=2}^{\infty} a_n$ is convergent.

(ii) The series $\sum_{n=1}^{\infty}(-1)^n \sqrt{\frac{3n+2}{4n-1}}$ is convergent.

(iii) If $0 \leq b_n \leq \frac{1}{\sqrt{n}}$ and $b_n \geq b_{n+1}$ for $n \geq 1$, then the series $\sum_{n=2}^{\infty}(-1)^n b_n$ is convergent.

A. (iii) only.

B. (i) only.

C. (ii) only.

D. (i) and (ii).

E. None

5. Which of the following statements are true?

(i) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty}(-1)^{n-1}a_n$ converges.

(ii) If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty}(-1)^{n-1}a_n$ converges.

(iii) If $\lim_{n\to\infty} n^2|a_n| = 3$, then $\sum_{n=1}^{\infty}(-1)^{n-1}a_n$ absolutely converges.

A. (i) and (iii).

B. (iii) only.

C. (ii) only.

D. (i) and (ii).

E. (ii) and (iii).

6. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent:

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n)}{2n^2-1}$

(ii) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{2n-1}$

A. (i) is absolutely convergent; (ii) is conditionally convergent.

B. (i) and (ii) are conditionally convergent.

C. (i) and (ii) are absolutely convergent.

D. (i) is conditionally convergent; (ii) is divergent.

E. (i) is conditionally convergent; (ii) is absolutely convergent.

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II) $\sum_{n=1}^{\infty}\left(a+\dfrac{1}{n}\right)^n$ diverges for $|a|>1$

III) If a sequence a_n is such that $-n-10 \leq \ln|a_n| \leq -n+1$ then $\sum_{n=1}^{\infty} a_n$ converges absolutely.

A. I, II and III are true

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A. $[-1,1]$

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9. Find the radius of convergence R of the series $\sum_{n=1}^{\infty} \frac{(x-10)^n}{5^n + n^2 + 6n}$

A. $R = 5$

B. $R = 4$

C. $R = 3$

D. $R = 2$

E. $R = 1$

10. The power series representation of the function $\frac{x}{1+x^3}$ is given by

A. $\sum_{n=1}^{\infty} (-1)^{n-1}\ x^{3n+1}$ for $|x| < 1$

B. $\sum_{n=1}^{\infty} (-1)^{n-1}\ x^{3n+3}$ for $|x| < 1$

C. $\sum_{n=1}^{\infty} (-1)^{n-1}\ x^{3n-2}$ for $|x| < 1$

D. $\sum_{n=1}^{\infty} (-1)^{n}\ x^{3n+2}$ for $|x| < 1$

E. $\sum_{n=1}^{\infty} (-1)^{n-1}\ x^{3n-1}$ for $|x| < 1$

11. Given that $\arctan(x^3) = \displaystyle\int_0^x \frac{3t^2}{1+t^6}\, dt$, the Maclaurin series of the function $\arctan(x^3)$ for $|x| < 1$ is given by

A. $\arctan(x^3) = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+5}}{n+2}$

B. $\arctan(x^3) = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+1}}{n+1}$

C. $\arctan(x^3) = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+1}}{2n+1}$

D. $\arctan(x^3) = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+3}}{2n+1}$

E. $\arctan(x^3) = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+3}}{6n+3}$

12. We know that

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We then conclude that the Taylor series of $\ln x$ centered at 3 and for $|\frac{x-3}{3}| < 1$ is given by

A. $\ln x = \ln 3 + \displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-2)^n}{n^{3n}}$

B. $\ln x = \ln 3 + \displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-3)^n}{n^2}$

C. $\ln x = \ln 3 + \displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-3)^n}{n}$

D. $\ln x = \ln 3 + \displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-3)^n}{n\, 3^{n+1}}$

E. $\ln x = \ln 3 + \displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-3)^n}{n\, 3^n}$