Student: ____________________ **Date:** ____________________	**Instructor:** Phil Mummert, Asaduzzaman Mohammad, Purdue Math, Jennifer Hobbs, Stephanie Foster, Sheng Zhang **Course:** MA166-102- Analytic Geom. And Calc. II- Spring 2020 **Assignment:** Exam3

1. Find the power series representation for g centered at 0 by differentiating or integrating the power series for f (perhaps more than once). Give the interval of convergence for the resulting series.

   $g(x) = \textbf{ln}\,(1-8x)$ using $f(x) = \dfrac{1}{1-8x}$

   Which of the following is the power series representation for g centered at 0?

   ○ **A.** $-\dfrac{1}{8}\sum_{k=1}^{\infty}\dfrac{8x^k}{k}$

   ○ **B.** $-\sum_{k=1}^{\infty}\dfrac{(8x)^k}{k}$

   ○ **C.** $-\dfrac{1}{8}\sum_{k=1}^{\infty}\dfrac{(8x)^k}{k}$

   ○ **D.** $-8\sum_{k=1}^{\infty}\dfrac{(8x)^k}{k}$

   The interval of convergence is ____________.
   (Simplify your answer. Type your answer in interval notation.)

2. Determine whether the following series converges absolutely, converges conditionally, or diverges.

$$\sum_{k=1}^{\infty}(-1)^k a_k = \sum_{k=1}^{\infty}\frac{(-1)^k k^2}{\sqrt{k^6+16}}$$

Find $\lim_{k\to\infty} a_k$. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** $\lim_{k\to\infty} a_k =$ ________
- **B.** The limit does not exist.

Now, let $\sum a_k$ denote $\sum_{k=1}^{\infty}\frac{(-1)^k k^2}{\sqrt{k^6+16}}$. What can be concluded from this result using the Divergence Test?

- **A.** The series $\sum a_k$ must converge.
- **B.** The series $\sum a_k$ must diverge.
- **C.** The series $\sum |a_k|$ must converge.
- **D.** The series $\sum |a_k|$ must diverge.
- **E.** The Divergence Test is inconclusive.

Are the terms of the sequence $|a_k|$ decreasing after some point?

- no
- yes

Let $\sum a_k$ denote $\sum_{k=1}^{\infty}\frac{(-1)^k k^2}{\sqrt{k^6+16}}$. What can be concluded from these results using the Alternating Series Test?

- **A.** The series $\sum a_k$ must diverge.
- **B.** The series $\sum a_k$ must converge.
- **C.** The series k^3 must converge.
- **D.** The series k^3 must diverge.
- **E.** The Alternating Series Test does not apply to this series.

Does the series $\sum |a_k|$ converge?

- **A.** no, as can be determined by the Limit Comparison Test
- **B.** yes, as can be determined by the Limit Comparison Test
- **C.** no, because of the Divergence Test
- **D.** no, because of properties of p-series
- **E.** yes, because of the Alternating Series Test
- **F.** yes, because of properties of p-series

Does the series $\sum a_k$ converge absolutely, converge conditionally, or diverge?

- A. The series diverges because $\sum |a_k|$ diverges.
- B. The series diverges because $\lim_{k\to\infty} a_k \neq 0$.
- C. The series converges conditionally because $\sum a_k$ converges but $\sum |a_k|$ diverges.
- D. The series converges absolutely because $\sum |a_k|$ converges.
- E. The series converges conditionally because $\sum |a_k|$ converges but $\sum a_k$ diverges.

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3. For the following telescoping series, find a formula for the nth term of the sequence of partial sums $\{S_n\}$. Then evaluate $\lim_{n\to\infty} S_n$ to obtain the value of the series or state that the series diverges.

$$\sum_{k=1}^{\infty} \frac{4}{(4k-3)(4k+1)}$$

$S_n =$ ____________

Select the correct choice and fill in any answer boxes in your choice below.

- A. $\sum_{k=1}^{\infty} \frac{4}{(4k-3)(4k+1)} =$ ____________ (Simplify your answer.)
- B. The series diverges.

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4. Use the Comparison Test or Limit Comparison Test to determine whether the following series converges.

$$\sum_{k=1}^{\infty} \sqrt{\frac{9k^6}{25k^8+3}}$$

Choose the correct answer below.

- A. The Limit Comparison Test with $\sum_{k=1}^{\infty} \frac{3}{5k}$ shows that the series diverges.
- B. The Limit Comparison Test with $\sum_{k=1}^{\infty} \frac{3}{5k}$ shows that the series converges.
- C. The Comparison Test with $\sum_{k=1}^{\infty} \frac{3}{5k}$ shows that the series diverges.
- D. The Comparison Test with $\sum_{k=1}^{\infty} \frac{3}{5k}$ shows that the series converges.

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5. Find the Taylor polynomials p_1, ..., p_4 centered at a = 0 for f(x) = **cos** (− 5x).

$p_1(x) =$ ____________

$p_2(x) =$ ____________

$p_3(x) =$ ____________

$p_4(x) =$ ____________

6. Use the Integral Test to determine whether the following series converges after showing that the conditions of the Integral Test are satisfied.

$$\sum_{k=1}^{\infty} \frac{5e^{k}}{1+e^{2k}}$$

Determine which of the necessary properties of the function that will be used for the Integral Test has. Select all that apply.

- ☐ **A.** The function f(x) is positive for x ≥ 1.
- ☐ **B.** The function f(x) is a decreasing function for x ≥ 1.
- ☐ **C.** The function f(x) is continuous for x ≥ 1.
- ☐ **D.** The function f(x) is negative for x ≥ 1.
- ☐ **E.** The function f(x) has the property that $a_k = f(k)$ for k = 1, 2, 3, ...
- ☐ **F.** The function f(x) is an increasing function for x ≥ 1.

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○ **A.** The series converges. The value of the integral $\int_{1}^{\infty} \frac{5e^{x}}{1+e^{2x}}\,dx$ is ____________.

(Type an exact answer.)

○ **B.** The series diverges. The value of the integral $\int_{1}^{\infty} \frac{5e^{x}}{1+e^{2x}}\,dx$ is ____________.

(Type an exact answer.)

○ **C.** The Integral Test does not apply to this series.

7. Use the Divergence Test to determine whether the following series diverges or state that the test is inconclusive.

$$\sum_{k=1}^{\infty} \frac{6k^8}{k!}$$

Choose the correct answer below.

- **A.** The series diverges because $\lim_{k\to\infty} \frac{6k^8}{k!} = 0$.
- **B.** The series diverges because $\lim_{k\to\infty} \frac{6k^8}{k!} \neq 0$.
- **C.** The series converges because $\lim_{k\to\infty} \frac{6k^8}{k!} = 0$.
- **D.** The series converges because $\lim_{k\to\infty} \frac{6k^8}{k!} \neq 0$.
- **E.** The Divergence Test is inconclusive.

8. **a.** Find the nth-order Taylor polynomials of the given function centered at the given point a, for n = 0, 1, and 2.
b. Graph the Taylor polynomials and the function.

$f(x) = \textbf{sin}\, x,\ a = \frac{\pi}{4}$

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a. Find the Taylor polynomial of order 0. Choose the correct answer below.

- **A.** $p_0(x) = 1$
- **B.** $p_0(x) = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$
- **C.** $p_0(x) = 0$
- **D.** $p_0(x) = \frac{\sqrt{2}}{2}$

Find the Taylor polynomial of order 1.

- **A.** $p_1(x) = \frac{\sqrt{2}}{2}$
- **B.** $p_1(x) = \left(x - \frac{\pi}{4}\right)$
- **C.** $p_1(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$
- **D.** $p_1(x) = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)^2$

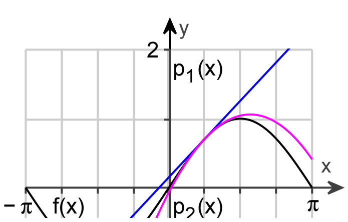
Find the Taylor polynomial of order 2.

- **A.**
- **B.** $p_2(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2$
- **C.** $p_2(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)$
- **D.** $p_2(x) = \left(x - \frac{\pi}{4}\right) + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)^2$

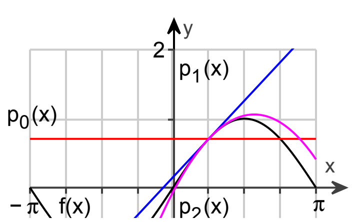
b. Choose the correct graph below.

$f(x) = \textbf{sin}\, x,\ a = \frac{\pi}{4}$

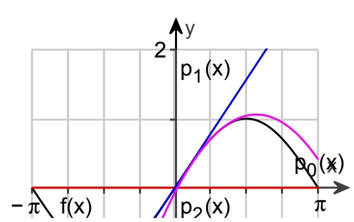
A.



B.



C.



9. Evaluate the series or state that it diverges.

$$\sum_{k=2}^{\infty} \left(\frac{3}{4}\right)^{2k}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** $\sum_{k=2}^{\infty} \left(\frac{3}{4}\right)^{2k} =$ ________ (Type an integer or a fraction.)
- **B.** The series diverges.

10. Use the Ratio Test to determine if the series converges.

$$\sum_{k=1}^{\infty} \frac{7(k!)^2}{8(2k)!}$$

Select the correct choice below and fill in the answer box to complete your choice.

- **A.** The series diverges because r = ________.
- **B.** The series converges because r = ________.
- **C.** The Ratio Test is inconclusive because r = ________.

11. Use the Root Test to determine whether the series converges.

$$\sum_{k=1}^{\infty} \left(\frac{k}{k+4}\right)^{2k^2}$$

Select the correct choice below and fill in the answer box to complete your choice.
(Type an exact answer in terms of e.)

- **A.** The series converges because $\rho =$ ________.
- **B.** The series diverges because $\rho =$ ________.
- **C.** The Root Test is inconclusive because $\rho =$ ________.

12. Find the interval of convergence of the series.

$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{n^2 5^n}$$

- A. $-4 \le x \le 6$
- B. $0 \le x \le 2$
- C. $-6 < x < 6$
- D.