MA166 — FINAL EXAM — FALL 2018 — DECEMBER 12, 2018
TEST NUMBER 11

INSTRUCTIONS:

1. **Do not open the exam booklet until you are instructed to do so.**
2. **Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
3. **MARK YOUR TEST NUMBER ON YOUR SCANTRON**
4. Once you are allowed to open the exam, make sure you have a complete test. There are 15 different test pages (including this cover page).
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers on this test booklet.
6. The exam has 25 problems and each one is worth 8 points. The maximum possible score is 200 points. No partial credit.
7. Do not leave the exam room during the first 20 minutes of the exam.
8. If you do not finish your exam in the first 40 minutes, you must wait until the end of the exam period to leave the room.
9. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

DON'T BE A CHEATER:

1. Do not give, seek or obtain any kind of help from anyone to answer questions on this exam. If you have doubts, consult only your instructor.
2. Do not look at the exam or scantron of another student.
3. Do not allow other students to look at your exam or your scantron.
4. You may not compare answers with anyone else or consult another student until after you have finished your exam, given it to your instructor and left the room.
5. Do not consult notes or books.
6. **Do not handle** phones or cameras, calculators or any electronic device until after you have finished your exam, given it to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: ____________________

STUDENT SIGNATURE: ____________________

STUDENT ID NUMBER: ____________________

SECTION NUMBER AND RECITATION INSTRUCTOR: ____________________

USEFUL FORMULAS

Trig Formulas:

$$\sin^2 x = \frac{1-\cos(2x)}{2}, \qquad \cos^2 x = \frac{1+\cos(2x)}{2}, \qquad \sec^2 x = 1+\tan^2 x$$

Useful Integrals:

$$\int \sec x\, dx = \ln|\sec x + \tan x| + C \quad \text{and} \quad \int \sqrt{1+x^2}\, dx = \frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\ln(x+\sqrt{1+x^2}) + C$$

Center of Mass:

$$\overline{x} = \frac{1}{A}\int_a^b x(f(x)-g(x))\, dx \qquad \text{and} \qquad \overline{y} = \frac{1}{A}\int_a^b \frac{1}{2}[(f(x))^2-(g(x))^2]\, dx$$

Arc Length, Surface Area and Volume:

Arc Length: $L = \displaystyle\int_a^b \sqrt{1+(f'(x))^2}\, dx$

Surface area: $S = 2\pi \displaystyle\int_a^b f(x)\sqrt{1+(f'(x))^2}\, dx$ or $S = 2\pi \displaystyle\int_a^b x\sqrt{1+(f'(x))^2}\, dx$

Volume by the washer method: $V = \pi \displaystyle\int_a^b (R^2(x) - r^2(x))\, dx$; $R(x)$ and $r(x)$ are the longer and shorter radii of the washer

Volume by cylindrical shells: $V = 2\pi \displaystyle\int_a^b x f(x)\, dx$

Maclaurin Series:

The geometric series: $\displaystyle\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, provided $|x| < 1$

The exponential function: $e^x = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$ for all x

Sine: $\sin x = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ for all x

Cosine: $\cos x = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ for all x

1. Find the angle (in radians) between the vectors $\vec{u} = \langle 1, 1, 0 \rangle$ and $\vec{v} = \langle 0, 1, 1 \rangle$.

A. $\pi/2$

B. $\pi/3$

C. $\pi/4$

D. 0

E. $\pi/6$

2. Find the area of the triangle with vertices $P(1, 1, 0)$, $Q(2, 1, 1)$ and $R(3, 1, 0)$.

A. 2

B. $\dfrac{1}{2}$

C. 1

D. $\dfrac{1}{3}$

E. $\dfrac{11}{6}$

3. The area of the region bounded by the curves $y = 3x$ and $y = x^2$. is

A. $\frac{4}{3}$

B. $\frac{2}{3}$

C. $\frac{8}{3}$

D. $\frac{5}{6}$

E. $\frac{9}{2}$

4. Find the volume of the solid obtained by rotating the region of the first quadrant bounded by the curves $y = x$ and $y = x^3$ about the x-axis.

A. $\frac{5\pi}{6}$

B. $\frac{4\pi}{3}$

C. $\frac{4\pi}{21}$

D. $\frac{8\pi}{21}$

E. $\frac{4\pi}{15}$

5. Find the volume of the solid obtained by rotating the region of the first quadrant bounded by $y = x^2 - x^3$ and $y = 0$ about the y-axis.

A. $\dfrac{\pi}{20}$

B. $\dfrac{\pi}{40}$

C. $\dfrac{\pi}{15}$

D. $\dfrac{\pi}{10}$

E. $\dfrac{\pi}{5}$

6. A force of 15 Newtons stretches a spring from its natural length of 5cm to 20cm. Find the total work done by stretching the spring from 25cm to 35cm.

A. 5 J

B. $\dfrac{5}{2}$ J

C. $\dfrac{5}{4}$ J

D. 6 J

E. $\dfrac{3}{2}$ J

7. Compute the integral $\int_0^{\frac{\pi}{2}} x \sin x \, dx$.

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. 1

D. $\frac{1}{4}$

E. $-\frac{1}{2}$

8. Compute the integral $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}} \, dx$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{8}$

D. $\frac{\pi}{3}$

E. $\frac{\pi}{6}$

9. Compute the integral $\displaystyle\int_1^2 \frac{3}{x^2+5x+4}\,dx$

A. $\ln\dfrac{5}{4}$

B. $\ln\dfrac{3}{5}$

C. $\ln\dfrac{9}{4}$

D. $\ln\dfrac{5}{3}$

E. $\ln\dfrac{11}{15}$

10. What can be said about the improper integral $\displaystyle\int_2^4 \frac{1}{\sqrt{x-2}}dx$?

A. It diverges

B. It converges and is equal to $\sqrt{2}$

C. It converges and is equal to 1

D. It converges and is equal to $\dfrac{1}{\sqrt{2}}$

E. It converges and is equal to $2\sqrt{2}$

11. Find the arc length of the curve $y = -\ln \cos x$ with $0 \le x \le \frac{\pi}{4}$. (Use one of the integrals given on page 2)

A. $\ln(1 + 2\sqrt{2})$

B. $\ln(2 + \sqrt{2})$

C. $\ln(1 + \sqrt{2})$

D. $1 + \ln(1 + \sqrt{2})$

E. $\dfrac{2}{3} + \ln(2 + \sqrt{2})$

12. After one makes a suitable substitution, the exact area of the surface obtained by rotating the curve $y = \cos x$, $0 \le x \le \dfrac{\pi}{2}$, about the x-axis is given by the following integral (the formula for the surface area is given on page 2 of the exam.)

A. $2\pi \displaystyle\int_0^1 \sqrt{1 + u^2}\, du$

B. $2\pi \displaystyle\int_0^1 u\sqrt{1 + u^2}\, du$

C. $2\pi \displaystyle\int_0^1 u^2\sqrt{1 + u^2}\, du$

D. $2\pi \displaystyle\int_0^1 u^2\sqrt{1 + u^4}\, du$

E. $2\pi \displaystyle\int_0^1 \sqrt{1 + u}\, du$

13. The area of the region of the first quadrant bounded by $y = x^3$ and $y = 1$ is equal to $\dfrac{3}{4}$. Find its center of mass (formulas for the center of mass can be found on page 2).

A. $\left(\dfrac{1}{3}, \dfrac{4}{9}\right)$

B. $\left(\dfrac{2}{3}, \dfrac{1}{7}\right)$

C. $\left(\dfrac{3}{8}, \dfrac{3}{5}\right)$

D. $\left(\dfrac{2}{5}, \dfrac{4}{7}\right)$

E. $\left(\dfrac{3}{5}, \dfrac{3}{4}\right)$

14. Compute the following $\displaystyle\lim_{n\to\infty}\left(\frac{3n^2 - 6n + 8}{n-1} - 3n\right)$

A. 2

B. -2

C. 3

D. -3

E. 0

15. The quantity $(\cos 2x)\sum_{n=1}^{\infty}(\tan x)^{2n}$, for $0 \leq x < \frac{\pi}{4}$ is equal to:

A. $\sin^2 x$

B. $\sin x(\sin x + \cos x)$

C. $\sin^2 x \cos^2 x$

D. $\tan^2 x$

E. $\sin 2x$

16. The series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{\frac{4}{3}}}$

A. converges by the integral test

B. diverges by the integral test

C. diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$

D. converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

E. diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^2}$

17. What can we say about the series

$$I)\sum_{n=2}^{\infty}\frac{e^{\frac{1}{n}}}{n},\qquad II)\sum_{n=2}^{\infty}(\frac{3}{2}+e^{-n})^{n}\quad \text{and } III)\sum_{n=1}^{\infty}\cos^{2}(\frac{1}{n})?$$

A. I, II and III diverge

B. I and III diverge, but II converges

C. I, II and III converge

D. I and II converge, but III diverges

E. I diverges, but II and III converge

18. Let a_n be a sequence such that $a_1 = 2$ and $\dfrac{a_{n+1}}{a_n} = \dfrac{2n+1}{n+5}$ and let b_n be another sequence such that $\displaystyle\lim_{n\to\infty}\frac{|b_n|}{|a_n|} = 3$. We can conclude that:

A. $\displaystyle\sum_{n=1}^{\infty} a_n$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ converge absolutely

B. $\displaystyle\sum_{n=1}^{\infty} a_n$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ diverge

C. $\displaystyle\sum_{n=1}^{\infty} a_n$ converges absolutely but we cannot say anything about the convergence of $\displaystyle\sum_{n=1}^{\infty} b_n$

D. $\displaystyle\sum_{n=1}^{\infty} a_n$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ converge conditionally

E. $\displaystyle\sum_{n=1}^{\infty} a_n$ converges absolutely and $\displaystyle\sum_{n=1}^{\infty} b_n$ diverges

19. Consider $S = \sum_{m=1}^{\infty} (-1)^{m-1} \frac{1}{m^4}$ and its partial sum $S_n = \sum_{m=1}^{n} (-1)^{m-1} \frac{1}{m^4}$. According to the alternating series estimation theorem, what is the smallest n such that $|S - S_n| < 5^4 \times 10^{-12}$?

A. $n = 250$

B. $n = 200$

C. $n = 199$

D. $n = 260$

E. $n = 249$

20. The interval of convergence of the series $\sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{10n+5}$ is

A. $(2, 4)$

B. $[2, 4)$

C. $[2, 4]$

D. $(2, 4]$

E. $[1, 2]$

21. Let $f(x) = \cos(x^3)$. Then $f^{(12)}(0)$ (the twelvth derivative of $\cos(x^3)$ at $x = 0$) is equal to: (the Maclaurin series for the $\cos x$ is given on page 2)

A. $f^{(12)}(0) = -\dfrac{12!}{10!}$

B. $f^{(12)}(0) = \dfrac{12!}{10!}$

C. $f^{(12)}(0) = -\dfrac{12!}{4!}$

D. $f^{(12)}(0) = \dfrac{12!}{8!}$

E. $f^{(12)}(0) = \dfrac{12!}{4!}$

22. Which of the following series is equal to $\displaystyle\int_0^1 \frac{\sin x}{x}\,dx$? (the Maclaurin series for the $\sin x$ is given on page 2)

A. $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{2}{(2n+1)(2n+1)!}$

B. $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{2}{(4n+1)(2n+1)!}$

C. $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{1}{(4n+1)(2n+1)!}$

D. $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{1}{(2n+1)(2n+1)!}$

E. $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{2}{(8n+1)(2n+1)!}$

23. Given $x(t) = t^3 + 1$ and $y(t) = t^2 - 1$, then $\dfrac{d^2y}{dx^2}$ for $t = 1$ is equal to

A. 1

B. $\dfrac{2}{3}$

C. $-\dfrac{2}{3}$

D. $-\dfrac{2}{9}$

E. $\dfrac{2}{9}$.

24. The Cartesian equation of the polar equation $r = 2\cos\theta + 4\sin\theta$ is

A. $x^2 + (y-2)^2 = 4$

B. $(x-1)^2 + (y-2)^2 = 5$

C. $(x+2)^2 + y^2 = 4$

D. $(x-1)^2 + (y+2)^2 = 5$

E. $x^2 + (y+2)^2 = 4$

25. Write the complex number $z = -1 + i\sqrt{3}$ in polar form with argument in $[0, 2\pi)$.

A. $z = 2e^{i\frac{3\pi}{4}}$

B. $z = 2e^{i\frac{5\pi}{4}}$

C. $z = 2e^{i\frac{2\pi}{3}}$

D. $z = 2e^{i\frac{\pi}{3}}$

E. $z = 2e^{i\frac{4\pi}{3}}$