NAME ______________________________

10-DIGIT PUID ______________________________

REC. INSTR. ______________ REC. TIME ______________

LECTURER ______________________________

INSTRUCTIONS:

1. There are 15 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. Also write your name at the top of pages 2–15.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet. No partial credit will be given, but if you show your work on the test booklet, it may be used in borderline cases.
4. No books, notes, calculators or any electronic devices may be used on this exam.
5. Each problem is worth 8 points. The maximum possible score is 200 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION NUMBER, put 0 in the first column and then enter the 3-digit section number. For example, for section 016 write 0016. Fill in the little circles.
   (c) On the bottom, under TEST/QUIZ NUMBER, write 01 and fill in the little circles.
   (d) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your 10–digit PUID, and fill in the little circles.
   (e) Using a #2 pencil, put your answers to questions 1–25 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your recitation instructor.

1. The equation of the sphere that passes through the origin and has its center at (3,2,-1) is:

   A. $(x+3)^2 + (y+2)^2 + (z-1)^2 = 14$

   B. $(x-3)^2 + (y-2)^2 + (z+1)^2 = 14$

   C. $x^2 + y^2 + z^2 = 14$

   D. $(x+3)^2 + (y+2)^2 + (z-1)^2 = 6$

   E. $(x-3)^2 + (y-2)^2 + (z+1)^2 = 6$

2. If $\vec{a} = \vec{i} + \vec{j}$ and $\vec{b} = \vec{i} + 2\vec{k}$, then the unit vector in the direction of their cross product $\vec{a} \times \vec{b}$ is:

   A. $\frac{1}{\sqrt{2}}(\vec{i} + \vec{j})$

   B. $\frac{1}{3}(2\vec{i} - 2\vec{j} - \vec{k})$

   C. $\frac{1}{3}(2\vec{i} + 2\vec{j} + \vec{k})$

   D. $\frac{1}{3}(2\vec{i} - \vec{j} - 2\vec{k})$

   E. $2\vec{i} - 2\vec{j} - \vec{k}$

3. The area of the triangle with vertices at the points $(1, 1, 1)$, $(2, -2, 2)$ and $(0, 0, 0)$ is:

A. 32

B. $2\sqrt{2}$

C. $2\sqrt{3}$

D. $4\sqrt{2}$

E. $4\sqrt{3}$

4. Find the volume of the solid obtained by rotating about the x-axis the region bounded by the curves $y = 2x^2$ and $y = 3x - x^2$.

A. $\frac{9}{10}\pi$

B. π

C. $\frac{1}{2}\pi$

D. $\frac{2}{3}\pi$

E. $\frac{3}{5}\pi$

5. A right inverted (i.e., upside down) circular conical tank of height 15 meters and base radius of 6 meters has its vertex at the bottom, and its axis vertical. If the tank is filled with water up to the 10-meter mark, then the work required to pump all the water out of the tank from the top is: (For computation, take the y-axis upwards along the axis of the tank and the origin at its vertex. The density of the water is 1000 kg/m^3 and the gravitational constant is g m/s^2)

A. $1000g \int_0^{10} \pi(\frac{2y}{5})^2(15-y)dy$

B. $1000g \int_0^{15} \pi(\frac{2y}{5})^2(10-y)dy$

C. $1000g \int_0^{10} \pi(\frac{2y}{5})^2 y dy$

D. $1000g \int_0^{15} \pi(\frac{2y}{5})^2 y dy$

E. $1000g \int_0^{10} \pi(\frac{2y}{5})^2(10-y)dy$

6. The base of a three-dimensional solid is the region in the xy-plane bounded by the circle $x^2 + y^2 = 1$. Parallel cross sections perpendicular to the base are squares. The volume of the solid is:

A. $\frac{3\pi}{16}$

B. $\frac{1}{2}$

C. $\frac{16}{3}$

D. 2

E. $\frac{16}{3}\pi$

7. Evaluate the following integral

$$\int_0^{\frac{\pi}{2}} \sin^4 x \; dx.$$

A. $\dfrac{\pi}{3}$

B. $\dfrac{\pi}{6}$

C. $\dfrac{3}{16}\pi$

D. $\dfrac{5}{8}\pi$

E. 2π

8. Evaluate the following integral

$$\int_3^4 \frac{8x-1}{x^2-x-2} dx.$$

A. $12\ln 2 - 3\ln 5$

B. $-\ln 2 + 3\ln 5$

C. $7\ln 2 - 2\ln 5$

D. $11\ln 2 - \ln 5$

E. $-3\ln 2 + 2\ln 5$

9. The proper form of the partial fractions for

$$\frac{x^4 - x^2 + 2}{x^2(x^4 - 16)}$$

is

A. $\dfrac{a}{x^2} + \dfrac{b}{x^2 - 4} + \dfrac{c}{x^2 + 4}$

B. $\dfrac{a}{x} + \dfrac{b}{x - 2} + \dfrac{c}{x + 2} + \dfrac{d}{x^2 + 4}$

C. $\dfrac{a}{x^2} + \dfrac{b}{x - 2} + \dfrac{c}{x + 2} + \dfrac{d}{x^2 + 4}$

D. $\dfrac{a}{x^2} + \dfrac{b}{x - 2} + \dfrac{c}{x + 2} + \dfrac{ex + f}{x^2 + 4}$

E. $\dfrac{a}{x} + \dfrac{b}{x^2} + \dfrac{c}{x - 2} + \dfrac{d}{x + 2} + \dfrac{ex + f}{x^2 + 4}$

(The letters a, b, c, d, e, f above represent some constants.)

10. Which of the following improper integrals converge ?

I. $\displaystyle\int_1^2 \frac{1}{\sqrt{2 - x}}dx$ II. $\displaystyle\int_{-1}^1 \frac{1}{x^3}dx$ III. $\displaystyle\int_2^\infty \frac{1}{x^2}dx$

A. II only

B. I and II only

C. II and III only

D. I and III only

E. All

11. Find the arc length of the curve

$$y = \frac{x^3}{3} + \frac{1}{4x}$$

on the interval $[1, 2]$.

A. $\frac{59}{24}$

B. $\frac{53}{24}$

C. $\frac{3\pi}{8}$

D. $\frac{3\pi}{4}$

E. $\tan^{-1}\frac{13}{16}$

12. Which of the following represents a formula for the area of the surface obtained by rotating the curve

$$y = \sqrt{1+4x}, \quad 1 \le x \le 5$$

about the x-axis ?

A. $2\pi \int_1^5 \sqrt{\frac{5+4x}{1+4x}}\, dx$

B. $2\pi \int_1^5 \sqrt{5+4x}\, dx$

C. $2\pi \int_1^5 \sqrt{1+4x}\, dx$

D. $2\pi \int_1^5 \sqrt{5+4x+x^2}\, dx$

E. $2\pi \int_1^5 x\sqrt{\frac{5+4x}{1+4x}}\, dx$

13. Consider the lamina bounded by the curves $y = e^x$, $y = 0$, $x = 0$ and $x = 1$, with density $\rho = 1$. The y-coordinate $\overline{y}$ of the center of mass of the lamina is

A. $\dfrac{1}{e-1}$

B. $e - 1$

C. $e + 1$

D. $\dfrac{e^2 - 1}{4}$

E. $\dfrac{e+1}{4}$

14. Which of the following series converges ?

I. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^{0.9}}$ II. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{2^n + 2}$ III. $\displaystyle\sum_{n=2}^{\infty} \arctan(n)$

A. I only

B. II only

C. III only

D. II and III only

E. None

15. Which of the following series converges ?

I. $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n-1}$ II. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ III. $\sum_{n=2}^{\infty} \sin\left(\frac{1}{n^2}\right)$

A. I and II only

B. II and III only

C. I and III only

D. All

E. None

16. Which of the following series CONDITIONALLY converges ?

I. $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$ II. $\sum_{n=2}^{\infty} (-1)^n \frac{n}{e^n}$ III. $\sum_{n=2}^{\infty} (-1)^n \frac{1}{e^{\frac{1}{n}}}$

A. I only

B. II only

C. III only

D. I and II only

E. None

17. Given that

$$\ln(1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots\right),$$

the Maclaurin series for the function $\ln\dfrac{1-x}{1+x}$ is given by:

A. $-2\left[x + \dfrac{x^3}{3} + \dfrac{x^5}{5} + \dfrac{x^7}{7} + \cdots\right]$

B. $\dfrac{1}{2}\left[x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots\right]$

C. $-\left[x + \dfrac{x^3}{3} + \dfrac{x^5}{5} + \dfrac{x^7}{7} + \cdots\right]$

D. $\dfrac{1}{2}\left[1 + \dfrac{x^2}{2} + \dfrac{x^4}{4} + \dfrac{x^6}{6} + \cdots\right]$

E. $2\left[1 - \dfrac{x^2}{2} + \dfrac{x^4}{4} - \dfrac{x^6}{6} + \cdots\right]$

18. The interval of convergence for the power series

$$\sum_{n=2}^{\infty} \frac{7^n}{n \ln n} x^n$$

is

A. $[-\frac{1}{7}, \frac{1}{7})$

B. $(-\frac{1}{7}, \frac{1}{7}]$

C. $(-\frac{1}{7}, \frac{1}{7})$

D. $(-\infty, \infty)$

E. $(-1, 1)$

19. Let $a = \lim_{n\to\infty} \left(\sqrt{n^2+n} - n\right)$ and $b = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$.

Then

A. $a = \frac{1}{2}$ and $b = e$

B. $a = \infty$ and $b = e$

C. $a = 0$ and $b = 1$

D. $a = \infty$ and $b = \infty$

E. $a = \infty$ and $b = 1$

20. Consider the parametric curve given by

$$x = \cos^3\theta, \quad y = \sin^3\theta, \quad 0 \le \theta \le 2\pi.$$

The visual picture of the curve is shown below:

Find the length of the curve.

A. 4

B. 5

C. 6

D. π

E. 4π

21. Consider the following curve given in polar coordinates:

$$r = 2\sin\theta + 2\cos\theta.$$

Which of the following is the correct description of the curve?

A. A circle centered at $(1, 1)$ with radius $\sqrt{2}$.

B. A circle centered at $(1, 1)$ with radius 2

C. A parabola with vertex $(-1, 1)$ and opens up towards the positive x-axis.

D. A parabola with vertex $(-1, 1)$ and opens up towards the negative x-axis.

E. An ellipse with center at $(-1, 1)$ with major and minor axises with length 4 and 2.

22. The number $\left(1 + i\sqrt{3}\right)^{10}$ is equal to

A. $2^9\left[1 + i\sqrt{3}\right]$

B. $2^{10}\left[1 + i\sqrt{3}\right]$

C. $\left[1 + i(3^5)\right]$

D. $-2^9\left[1 + i\sqrt{3}\right]$

E. $2^9\left[1 - i\sqrt{3}\right]$

23. Using the method of cylindrical shells, find the formula for the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.

A. $\displaystyle\int_0^{\frac{1}{4}} \pi y^2 dy$

B. $\displaystyle\int_0^1 \pi(x - x^2)^2 dx$

C. $\displaystyle\int_0^1 2\pi x(x - x^2) dx$

D. $\displaystyle\int_0^1 2\pi(2 - x)(x - x^2) dx$

E. $\displaystyle\int_0^1 2\pi(2-x)(x-x^2)dx + \int_3^4 2\pi(x-2)(x-3)(4-x)dx$

24. Evaluate the following integral

$$\int_1^2 \frac{1}{\sqrt{4x - x^2}} dx$$

A. $\frac{1}{6}$

B. 3π

C. $\frac{\pi}{3}$

D. 2

E. $\frac{\pi}{6}$

25. Using the Maclaurin series

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!}x^n = 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$$

we have the formula

$$e^{-0.1} = 1 - \frac{1}{1!}0.1 + \frac{1}{2!}0.1^2 - \frac{1}{3!}0.1^3 + \cdots .$$

Instead of the infinite sum above, we would like to use the partial sum

$$S_k = 1 - \frac{1}{1!}0.1 + \frac{1}{2!}0.1^2 - \frac{1}{3!}0.1^3 + \cdots + (-1)^k \frac{1}{k!}0.1^k$$

as an approximation for $e^{-0.1}$.

What is the least number k which guarantees $|e^{-0.1} - S_k| < 10^{-5}$, using the estimation theorem for the alternating series?

A. 2

B. 3

C. 4

D. 5

E. 6