MA 16600
FINAL EXAM INSTRUCTIONS
VERSION 01
May 7, 2015

Your name ______________________ Your TA's name ______________________

Student ID # ________________ Section # and recitation time ___________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet.

7. Write down YOUR NAME and TA's NAME on the exam booklet.

8. There are 25 questions, each worth 8 points. The total is $8 \times 25 = 200$. Blacken your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets. Turn in both the scantron sheets and the question sheets when you are finished.

9. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

10. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

11. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

12. If you finish the exam before 9:55 AM, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 9:55 AM, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ________________________________

STUDENT SIGNATURE: ____________________________

Questions

1. If the sphere given by $x^2 + y^2 + z^2 + 8x - 4y + 6z = C$ passes through the point $(-1, 2, 1)$, then its radius is:

A. 1

B. 2

C. 3

D. 4

E. 5

2. If the two vectors $\langle y, 3, 2\rangle$ and $\langle y, -4, -2y\rangle$ are perpendicular to each other, then the value of y is:

A. $y = -2$ or $y = 6$

B. $y = 2$ or $y = -6$

C. $y = 0$

D. $x = -2\sqrt{2} - 1$ or $y = -2\sqrt{2} + 1$

E. There is no such value for y.

3. The area of the triangle with vertices $(0,0,0)$, $(2,-1,2)$ and $(4,1,3)$ is:

A. $\frac{\sqrt{41}}{2}$

B. $\frac{\sqrt{65}}{2}$

C. $2\sqrt{65}$

D. $\frac{\sqrt{61}}{2}$

E. $\frac{2\sqrt{61}}{3}$

4. The region bounded by the graphs of functions $y = 3x$, $y = 0$, and $x = 3$ is rotated about the y-axis.

The volume of the solid is given by the integral:

A. $\int_0^3 \pi \left(9 - \frac{y^2}{9}\right) dy$

B. $\int_0^3 \pi \left(3 - 9y^2\right) dy$

C. $\int_0^3 2\pi \left(3 - 9y^2\right) dy$

D. $\int_0^9 \pi \left(9 - \frac{y^2}{9}\right) dy$

E. $\int_0^9 2\pi \left(9 - \frac{y^2}{9}\right) dy$

5. Evaluate $\int_1^e x^2 \ln x dx$.

A. $\dfrac{e^3+1}{3}$

B. $\dfrac{e^3}{3}$

C. $\dfrac{e^2+e}{9}$

D. $\dfrac{2e^3}{9}$

E. $\dfrac{2e^3+1}{9}$

6. The work required to stretch a spring 6 ft beyond its natural length is 28 ft-lb. How much work is needed to stretch the spring 3 ft beyond its natural length ?

A. 7 ft-lb.

B. $\dfrac{9}{2}$ ft-lb.

C. $\dfrac{28}{3}$ ft-lb.

D. $\dfrac{14}{9}$ ft-lb.

E. $\dfrac{21}{4}$ ft-lb.

7. A trigonometric substitution will convert the integral

$$\int_1^3 \sqrt{x^2+2x-3}\; dx$$

into which of the following ?

A. $\displaystyle\int_0^{\pi/3} 4\sec\theta\tan\theta d\theta$

B. $\displaystyle\int_0^{\pi/3} 4\sec\theta\tan^2\theta d\theta$

C. $\displaystyle\int_0^{\pi/3} 2\sec\theta\tan^3\theta d\theta$

D. $\displaystyle\int_0^{\pi/6} 2\sec\theta\tan^2\theta d\theta$

E. $\displaystyle\int_0^{\pi/3} \sec\theta\tan\theta d\theta$

8. Compute the integral

$$\int_{-\pi/2}^{0} \cos^2 x \sin^5 x\; dx.$$

A. $\frac{17}{35}$

B. $\frac{-8}{105}$

C. $\frac{3}{35}$

D. $\frac{-5}{21}$

E. $\frac{-31}{105}$

9. Compute the integral

$$\int_0^{\frac{\pi}{4}} (\sec x)^4 (\tan x)^2 \; dx.$$

A. $\frac{8}{3}$
B. $\frac{3}{2}$
C. $\frac{4}{5}$
D. $\frac{10}{3}$
E. $\frac{8}{15}$

10. Compute the integral

$$\int_{1/3}^{2/3} \frac{xdx}{\sqrt{9x^2-1}}.$$

A. $\frac{\sqrt{3}}{4}$
B. $\frac{4}{9}$
C. $\frac{\sqrt{3}}{9}$
D. $\frac{\sqrt{3}}{3}$
E. $\frac{1}{18}$.

11. When one makes a suitable trigonometric substitution, the integral

$$\int \frac{x^3}{\sqrt{4-x^2}}dx$$

becomes:

A. $4\int \sin^2\theta \tan\theta d\theta$

B. $8\int \sin^3\theta d\theta$

C. $8\int \tan^3\theta \sec\theta d\theta$

D. $8\int \tan^4\theta d\theta$

E. $\int \sin^3\theta d\theta$

12. Compute the area of the region enclosed by the graphs $x = 3-y^2+2y$ and $x = y^2-2y+3$.

A. $\frac{8}{5}$

B. $\frac{8}{3}$

C. $\frac{7}{3}$

D. $\frac{13}{3}$

E. $\frac{14}{5}$

13. If $\int \frac{2}{x^2(x+1)}dx = \alpha \ln|x| + \beta\frac{1}{x} + \gamma \ln|x+1| + C$ for some numbers α, β, γ and the integral constant C, what is the value of γ?

A. 3

B. 2

C. $\frac{2}{3}$

D. -3

E. -2

14. Given the following three series

I. $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n^3}\right)$

II. $\sum_{1}^{\infty} \frac{\ln n}{n^2}$

III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$,

we conclude that:

A. I and II diverge but III converges

B. I and III diverge but II converges

C. All three converge

D. All three diverge

E. II and III converge but I diverges

15. The proper from of the partial fraction decomposition of $\dfrac{3x+2}{(x^2+4)(x^4-16)}$ is:

A. $\dfrac{A}{x^2+4}+\dfrac{B}{x^4-16}$

B. $\dfrac{A}{x^2+4}+\dfrac{Bx+C}{x^4-16}$

C. $\dfrac{A}{(x-2)}+\dfrac{B}{x+2}+\dfrac{Cx+D}{x^2+4}$

D. $\dfrac{A}{(x-2)}+\dfrac{B}{x+2}+\dfrac{Cx+D}{x^2+4}+\dfrac{Ex+F}{(x^2+4)^2}$

E. $\dfrac{A}{(x-2)}+\dfrac{B}{x+2}+\dfrac{C}{x^2+4}$

16. Compute $\displaystyle\sum_{n=0}^{\infty}\frac{(-2)^{n+1}}{3^n}$

A. $\frac{4}{5}$

B. $\frac{12}{5}$

C. $-\frac{6}{5}$

D. $\frac{6}{5}$

E. -12

17. An equivalent expression for the complex number $\dfrac{2+5i}{3-i}$ is:

A. $\frac{2+25i}{3+i}$

B. $\frac{17+i}{5}$

C. $\frac{2+17i}{10}$

D. $\frac{1+17i}{9+i}$

E. $\frac{1+17i}{10}$

18. Find the interval of convergence for the power series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{\sqrt{n}} x^n$.

A. $[-\dfrac{1}{3}, \dfrac{1}{3})$

B. $(-\dfrac{1}{3}, \dfrac{1}{3}]$

C. $(-\dfrac{1}{3}, \dfrac{1}{3})$

D. $[-\dfrac{1}{3}, \dfrac{1}{3}]$

E. $(-\infty, \infty)$

19. The Maclaurin series for the function $f(x) = \dfrac{x}{(1+x^2)^2}$ is:

A. $\sum_{n=1}^{\infty}(-1)^n x^{2n}$

B. $\sum_{n=1}^{\infty}(-1)^n 2n x^{2n-1}$

C. $\sum_{n=1}^{\infty}(-1)^n n x^{2n-1}$

D. $\sum_{n=1}^{\infty}(-1)^{n+1} n x^{2n-1}$

E. $\sum_{n=1}^{\infty}\dfrac{(-1)^n}{2n+1}x^{2n+1}$

HINT: What is the Maclaurin series for $g(x) = \dfrac{1}{1+x^2}$?

20. Given the parametric equations

$$\begin{cases} x &= 3t^2 \\ y &= t^2 + 2t, \end{cases}$$

find the second derivative $\dfrac{d^2y}{dx^2}$ at the point $(3,3)$.

A. $\frac{1}{3}$

B. $-\frac{1}{3}$

C. $-\frac{1}{486}$

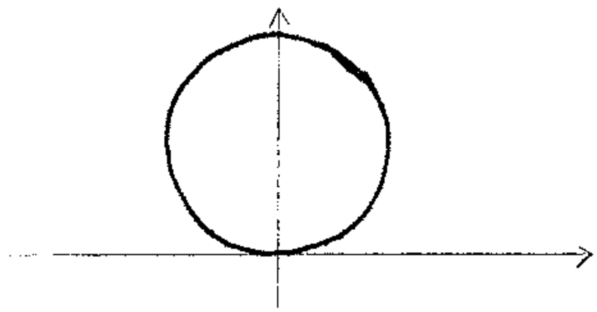
D. $\frac{1}{18}$

E. $-\frac{1}{18}$

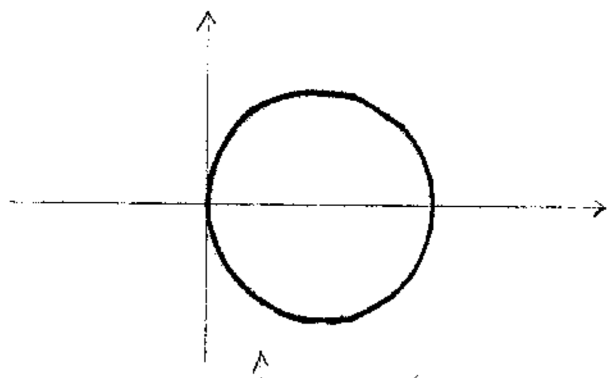
21. Choose the picture that best describes the curve given by the equation in polar coordinates

$$r = \sin 2\theta.$$

A.



B.



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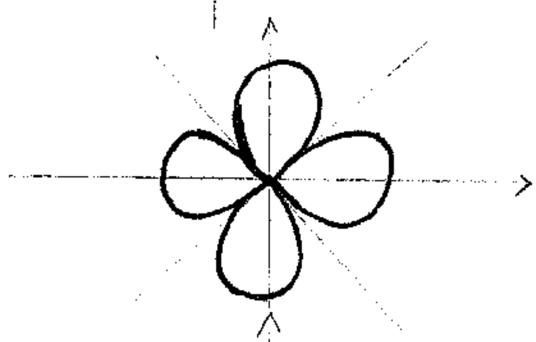
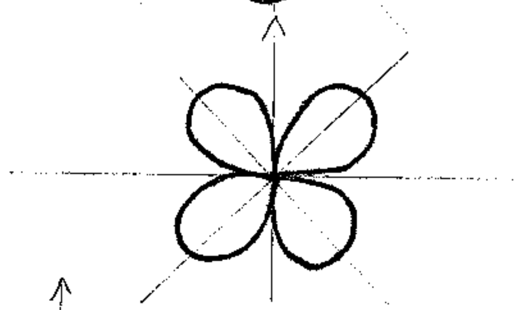
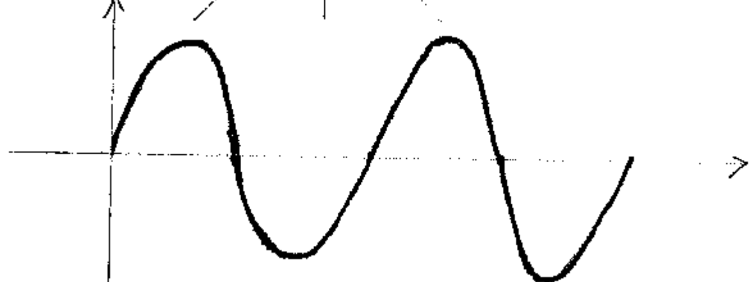


D.



E.



22. Compute the length of the curve given by the parametric equations

$$\begin{cases} x &= e^{2t} + e^{-2t} \\ y &= 2 - 4t \end{cases} \quad \text{for } 0 \le t \le 1.$$

A. $(e + e^{-1})^2$

B. $e^2 + e^{-2} + 4$

C. $\frac{1}{2}(e^2 + e^{-2})$

D. $e^2 - e^{-2}$

E. $\frac{1}{4}(e^2 - e^{-2} + 4)$

23. Using the power series

$$\ln(1 + x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$$

and the Estimation Theorem for the Alternating Series, we conclude that the least number of terms in the series needed to approximate $\ln(2)$ with error $< 0.3 \times 10^{-2}$ is:

A. 333

B. 334

C. 99

D. 100

E. 101

24. Let z be the following complex number

$$z = \sqrt{3} + i.$$

Compute z^9.

A. $z = 2^9$

B. $z = 2^9 i$

C. $z = -2^9 i$

D. $z = 2^9 \left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)$

E. $z = 2^9 \left(\frac{\sqrt{3}}{2} - \frac{1}{2} i\right)$

25. The center of mass $(\overline{x}, \overline{y})$ of the region bounded by $y = x^2$ and $x = y^2$ with uniform density ρ is:

A. $(\frac{20}{9}, \frac{20}{9})$

B. $(\frac{9}{20}, \frac{9}{20})$

C. $(\frac{9}{20}, \frac{9}{10})$

D. $(\frac{9}{10}, \frac{9}{20})$

E. $(\frac{1}{2}, \frac{1}{2})$

HINT: Use the following computation of the integral if needed:

$$\begin{aligned}
\int_0^1 (\sqrt{x} - x^2)\, dx &= \tfrac{1}{3} \\
\int_0^1 x(\sqrt{x} - x^2)\, dx &= \tfrac{3}{20} \\
\int_0^1 (x - x^4)\, dx &= \tfrac{3}{10}
\end{aligned}$$