MA 16600
FINAL EXAM INSTRUCTIONS
VERSION 01
May 3, 2017

Your name ______________________ Your TA's name ______________________

Student ID # ________________ Section # and recitation time ___________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet.

7. Write down YOUR NAME and TA's NAME on the exam booklet.

8. There are 25 questions, each worth 8 points. The total is $8 \times 25 = 200$. Blacken your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets.

9. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

10. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

11. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet (question sheets).

12. If you finish the exam before 8:55 PM, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 8:55 PM, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets (question sheets).

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ____

STUDENT SIGNATURE: ____

Questions

1. The area of the triangle with vertices $(1, 2, 1)$, $(1, 3, 1)$ and $(2, 1, 2)$ is:

A. 2

B. $\sqrt{2}$

C. 3

D. $\dfrac{\sqrt{2}}{2}$

E. $\dfrac{\sqrt{3}}{2}$

2. The vectors $\vec{a} = \langle 1, 1/2, \alpha \rangle$ and $\vec{b} = \langle \beta, 0, 2 \rangle$ are orthogonal to each other, with $|\vec{a}| = 2$ and $\beta > 0$. The value of β is:

A. $\sqrt{6}$

B. $\sqrt{\dfrac{17}{3}}$

C. $\sqrt{11}$

D. $\sqrt{7}$

E. There is no such value for β that satisfies the given conditions.

3. Consider the region bounded by the curves $y = x^2$ and $x = y^2$.

The volume of the solid obtained by rotating the region about the line $y = 1$ is:

A. $\dfrac{\pi}{4}$

B. 25π

C. $\dfrac{11}{30}\pi$

D. $\dfrac{6}{\pi}$

E. $\dfrac{\pi^2}{6}$

4. If we use the method of cylindrical shells, then the volume of the solid obtained by rotating the region bounded by the curves

$$y = e^{-x^2}, y = 0, x = 3, x = 5$$

about the line $x = 2$ is expressed by the integration:

A. $\displaystyle\int_3^5 2\pi(x-2)e^{-x^2}\,dx$

B. $\displaystyle\int_3^5 2\pi x e^{-x^2}\,dx$

C. $\displaystyle\int_0^1 2\pi y\sqrt{-\ln y}\,dy$

D. $\displaystyle\int_{-5}^5 2\pi x e^{-x^2}\,dx$

E. $\displaystyle\int_0^5 2\pi(x+2)e^{-x^2}\,dx$

5. The average of the function $f(x) = \cos^4 x \sin x$ on the interval $[0, \pi]$ is:

A. $\dfrac{4}{\pi}$

B. $\dfrac{3}{2\pi}$

C. $\dfrac{2}{5\pi}$

D. $\dfrac{3}{5\pi}$

E. $\dfrac{1}{\pi}$

6. A 12 ft chain weighs 15 lbs and hangs from a ceiling. The work done to lift the lower end so that it is level with the upper end is given by the formula:

A. $\displaystyle\int_0^6 \frac{15}{12} \cdot 2(6 - x)\ dx$

B. $\displaystyle\int_0^6 \frac{15}{12}(6 - x)\ dx\ dx$

C. $\displaystyle\int_0^{12} \frac{15}{12}(12 - x) dy$

D. $\displaystyle\int_0^{15} \frac{12}{15}(15 - x)\ dx$

E. $\displaystyle\int_0^6 \frac{12}{15}(15 - x)\ dx$

7. The length of the curve

$$y = \frac{1}{2}x^2 - \frac{1}{4}\ln(x), \quad 1 \le x \le 2$$

is:

A. $\frac{1}{4} + \frac{1}{2}\ln 2$

B. $\frac{3}{2} + \frac{1}{4}\ln 2$

C. $4 + 5\ln 3$

D. $2 + \frac{1}{8}\ln 2$

E. $7 + \ln 3$

8. The Maclaurin series for the function $f(x)$ is given by the formula

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{3n^2(n+5)}.$$

The value of $f^{(5)}(0)$ (the 5-th derivative of f evaluated at $x = 0$) is:

A. $-\frac{4}{25}$

B. $\frac{4}{25}$

C. $-\frac{1}{750}$

D. $\frac{1}{750}$

E. 0

9. After an appropriate substitution, the integral

$$\int_0^{\frac{\pi}{3}} \tan^3(x)\sec(x)\ dx$$

is computed to be:

A. $2\sqrt{3}$
B. $\frac{2}{3}$
C. $\frac{4}{3}$
D. $3\sqrt{2}$
E. $\frac{1}{2}$

10. Compute the integral

$$\int_{\pi/2}^{3\pi/4} \frac{\cos^3(\theta)}{\sin(\theta)}\ d\theta.$$

A. $\frac{3}{2} + \frac{1}{2}\ln 2$
B. $\frac{1}{4} - \frac{1}{2}\ln 2$
C. $\frac{3}{2} - \frac{1}{2}\ln 2$
D. $2 + \ln 2$
E. $5 + 3\ln 2$

11. Evaluate the integral

$$\int_1^e \frac{2\ln(x)}{x^2}.$$

A. $\dfrac{e^2+2}{e}$

B. $\dfrac{5-3e^2}{e}$

C. $\dfrac{2e^2-4}{e}$

D. $\dfrac{2e-4}{e}$

E. $\dfrac{2e+4}{e}$

12. Evaluate the integral

$$\int_0^1 \frac{8x-4}{x^2-2x+2}\,dx.$$

A. $\dfrac{\pi}{4}$

B. $-4\ln 2$

C. $\dfrac{\pi}{4}-\ln 2$

D. $4\ln 2-\pi$

E. $\pi-4\ln 2$

13. The appropriate trigonometric substitution will convert the following integral

$$\int_2^5 \frac{dt}{\sqrt{t^2 - 4t + 13}}$$

into:

A. $\displaystyle\int_0^{\pi/3} \cos(\theta)\ d\theta$

B. $\displaystyle\int_0^{\pi/3} \sin(\theta)\ d\theta$

C. $\displaystyle\int_0^{\pi/3} \sec(\theta)\ d\theta$

D. $\displaystyle\int_0^{\pi/4} \sec(\theta)\ d\theta$

E. $\displaystyle\int_0^{\pi/6} \sin(\theta)\ d\theta$

14. The series $\displaystyle\sum_{n=0}^{\infty} 3\left(\frac{-1}{4}\right)^n$ is:

A. divergent

B. convergent and its value is $\frac{1}{4}$

C. convergent and its value is $\frac{12}{5}$

D. convergent and its value is $-\frac{3}{5}$

E. convergent by the Alternating Series Test, but we can not evaluate its value.

15. The series $\sum_{n=0}^{\infty}(-1)^n \frac{\arctan(n)}{n^3+1}$ is:

A. absolutely convergent

B. conditionally convergent

C. divergent because $\lim_{n\to\infty}(-1)^n \frac{\arctan(n)}{n^3+1} \neq 0$

D. divergent by the Ratio Test

E. neither convergent nor divergent because $\lim_{n\to\infty} \frac{\arctan(n)}{n^3+1} = \frac{\infty}{\infty} = 1.$

16. The area of the surface obtained by rotating the curve

$$y = x^2 + 1, \ \ 0 \leq x \leq 3$$

is given by the formula:

A. $\int_0^3 2\pi(x^2+1)\sqrt{1+4x^2}\ dx$

B. $\int_0^3 \pi(x^2+1)^2 3\ dx$

C. $\int_0^3 \pi(x^2+1)^2\sqrt{1+4x^2}\ dx$

D. $\int_0^3 2\pi x(x^2+1)\ dx$

E. $\int_1^{10} \pi(1-y)\ dy$

17. The radius R and the interval I of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{2^n}{n} x^n$$

are:

A. $R = \frac{1}{2}, \quad I = \left(-\frac{1}{2}, \frac{1}{2}\right)$

B. $R = \frac{1}{2}, \quad I = \left[-\frac{1}{2}, \frac{1}{2}\right)$

C. $R = \frac{1}{2}, \quad I = \left(-\frac{1}{2}, \frac{1}{2}\right]$

D. $R = 1, \quad I = (-1, 1)$

E. $R = \infty, \quad I = (-\infty, \infty)$

18. If the curve is defined by the parametric equations

$$\begin{cases} x &= t^2 \\ y &= t^2 + t, \end{cases}$$

then the value of $\frac{d^2y}{dx^2}$ at the point $(1, 0)$ is:

A. $\frac{1}{4}$

B. $-\frac{1}{4}$

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

E. $\frac{1}{3}$

19. An equivalent expression for the complex number

$$\frac{3+8i}{4-2i}$$

is:

A. $-\dfrac{4-38i}{20}$

B. $-\dfrac{-4+38i}{20}$

C. $-\dfrac{28+26i}{20}$

D. $\dfrac{73}{20}$

E. $-\dfrac{13}{4}+\dfrac{7}{2}i$

20. Consider the following two complex numbers

$$\begin{cases} z &= 5+5\sqrt{3}i \\ w &= 2\sqrt{3}+2i. \end{cases}$$

Then z/w in polar coordinates is given by:

A. $10\left\{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)\right\}$

B. $10\left\{\cos\left(\frac{\pi}{3}\cdot\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{3}\cdot\frac{\pi}{6}\right)\right\}$

C. $\frac{5}{2}\left\{\cos\left(\frac{\pi}{3}-\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{3}-\frac{\pi}{6}\right)\right\}$

D. $\frac{2}{5}\left\{\cos\left(\frac{\pi}{6}-\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{6}-\frac{\pi}{3}\right)\right\}$

E. $\frac{5}{2}\left\{\cos\left(\frac{\pi}{3}/\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{3}/\frac{\pi}{6}\right)\right\}$

21. Choose the picture which best depicts the curve defined by the following equation in polar coordinates

$$r = \sin(3\theta), \;\; 0 \leq \theta \leq \pi.$$

A.

B.

C.

D.

E.

22. Which of the following series converge ?

(I) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2}$ (II) $\displaystyle\sum_{n=1}^{\infty} \frac{5n^n}{n!}$ (III) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4^n - 1}$ (IV) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt[3]{n}}$

A. (I) only

B. (III) and (IV) only

C. (I), (III), and (IV) only

D. All (I), (II), (III), and (IV)

E. (II) only

23. The Maclaurin series for

$$\int \frac{e^{2x} - 1}{x}\, dx$$

is given by

A. $\displaystyle\sum_{n=1} \frac{2^n x^n}{n!} + C$

B. $\displaystyle\sum_{n=1} \frac{2^n x^{n+1}}{(n+1)!} + C$

C. $\displaystyle\sum_{n=1} \frac{2^{n+1} x^n}{n!n} + C$

D. $\displaystyle\sum_{n=1} \frac{2^n x^n}{(n+1)!} + C$

E. $\displaystyle\sum_{n=1} \frac{2^n x^n}{n!n} + C$

Note: The letter "C" above represents the integration constant.

24. The lamina of uniform density bounded by the curves

$$y = \sqrt{x}, y = 0, x = 4$$

has the x-coordinate $\bar{x}$ of the center of mass equal to:

A. 1

B. 3

C. $\frac{5}{2}$

D. $\frac{12}{5}$

E. $\frac{6}{7}$

25. Compute the integral

$$\int_2^3 \frac{x+1}{x(x-1)}\, dx.$$

A. $\ln 5$

B. $\ln\left(\frac{5}{6}\right)$

C. $\ln\left(\frac{8}{3}\right)$

D. $\frac{9}{4}$

E. $\frac{9}{5}$