MA 16600
FINAL EXAM INSTRUCTIONS
VERSION 01
May 1, 2018

Your name ______________________ Your TA's name ______________________

Student ID # ________________ Section # and recitation time ___________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit **SECTION NUMBER**.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–25. **All the answers must be marked on the scantron sheet.** In case what is marked on the scantron sheet is different from what is marked on the exam booklet, we compute the final score based upon what is marked on the scantron sheet.

8. While marking all your answers on the scantron sheet, you should **show your work on the exam booklet**. In case of a suspicious activity of academic dishonesty and/or under certain circumstances, we require that the correct answer on the scantron sheet must be supported by the work on the exam booklet.

9. There are 25 questions, each worth 8 points. The maximum possible score is $8 \times 25 = 200$ points.

10. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

11. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

12. If you finish the exam before 2:55, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 2:55, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ____

STUDENT SIGNATURE: ____________________________________

Questions

1. Given the points

$$\begin{cases} P(1,2,1), \\ Q(-1,3,-2), \\ R(2,-3,4), \end{cases}$$

set

$$\begin{cases} \vec{a} &=& \overrightarrow{PR}, \\ \vec{b} &=& \overrightarrow{PQ}. \end{cases}$$

Find the projection $\text{Proj}_{\vec{a}}\vec{b}$ of the vector $\vec{b}$ onto the vector $\vec{a}$.

A. $\dfrac{16}{\sqrt{35}}\left(\dfrac{\vec{a}}{|\vec{a}|}\right)$

B. $\dfrac{-16}{\sqrt{35}}\left(\dfrac{\vec{a}}{|\vec{a}|}\right)$

C. $-16\left(\dfrac{\vec{a}}{|\vec{a}|}\right)$

D. $\dfrac{-16}{\sqrt{14}}\left(\dfrac{\vec{a}}{|\vec{a}|}\right)$

E. $\dfrac{16}{\sqrt{14}}\left(\dfrac{\vec{a}}{|\vec{a}|}\right)$

2. Find the area of the parallelogram formed by the two vectors $\vec{i} - \vec{j}$ and $\vec{i} + \vec{j} + \vec{k}$.

A. 2

B. $\sqrt{6}$

C. $\sqrt{3}$

D. $\sqrt{5}$

E. 0

3. Evaluate the integral

$$\int_0^1 \frac{1}{\sqrt{4-3x^2}}\,dx.$$

A. $\dfrac{\pi}{3\sqrt{3}}$

B. $\dfrac{\pi}{6}$

C. $\dfrac{\pi}{6\sqrt{3}}$

D. $\dfrac{\pi}{3}$

E. $\dfrac{1}{3}$

4. Compute the average of the function $f(x) = \tan^3 x$ over the interval $[0, \pi/4]$.
HINT: $\tan^3 x = (\sec^2 x - 1)\tan x$

A. $\dfrac{1-\ln 2}{2}$

B. $\dfrac{2(1-\ln 2)}{\pi}$

C. $\dfrac{4}{\pi}$

D. $\dfrac{\pi}{3}$

E. $\dfrac{1}{3}$

5. A spherical tank 6 feet in diameter is half full of water (density: 62.5 lb/ft^3). The work required to pump all the water in the tank to the top of the tank is (let the origin be at the center of the tank and the y–axis point upward):

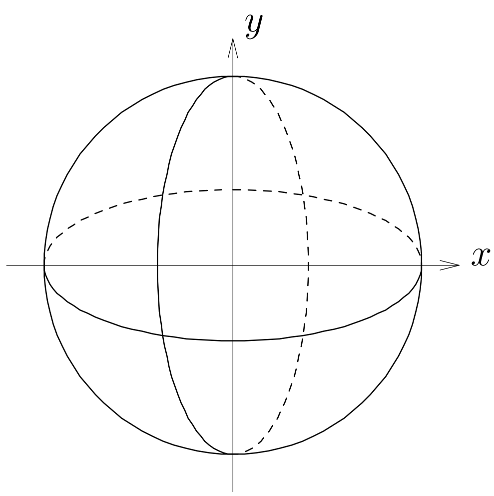
A. $\displaystyle\int_{-3}^{0} 62.5(3-y)\pi(9-y^2)\;dy$ ft-lb

B. $\displaystyle\int_{-3}^{0} 62.5(3+y)\pi(9-y^2)\;dy$ ft-lb

C. $\displaystyle\int_{-3}^{3} 62.5(6-y)\pi(9-y^2)\;dy$ ft-lb

D. $\displaystyle\int_{-3}^{3} 62.5(3+y)\pi\sqrt{9-y^2}\;dy$ ft-lb

E. $\displaystyle\int_{0}^{3} 62.5(3-y)\pi\sqrt{9-y^2}\;dy$ ft-lb



6. Compute the indefinite integral

$$\int \frac{dx}{x^2 - 4x + 3}.$$

A. $\dfrac{1}{2}\dfrac{\ln|x-1|}{\ln|x-3|} + \mathrm{C}$

B. $\dfrac{1}{2}\dfrac{\ln|x-3|}{\ln|x-1|} + \mathrm{C}$

C. $\dfrac{1}{2}\ln\left|\dfrac{x-3}{x-1}\right| + \mathrm{C}$

D. $\dfrac{1}{2}\ln\left|\dfrac{x-1}{x-3}\right| + \mathrm{C}$

E. $\ln|x^2 - 4x + 3| + \mathrm{C}$

7. $\displaystyle\int_0^{\pi/2} \sin^3 x \, dx =$

A. $\dfrac{1}{4}$

B. $\dfrac{1}{3}$

C. $\dfrac{\pi}{2} + \dfrac{1}{3}$

D. $\dfrac{2}{3}$

E. $\dfrac{4}{3}$

8. Which of the following improper integrals converges?

I. $\displaystyle\int_0^1 \frac{dx}{\sqrt{x}}$ II. $\displaystyle\int_1^\infty \frac{dx}{1+x^2}$ III. $\displaystyle\int_0^1 \frac{dx}{x^2}$

A. I and II only.

B. II only.

C. I only.

D. I, II, and III.

E. None of them.

9. Consider the following two complex numbers

$$\begin{cases} z &= 1+\sqrt{3}i \\ w &= 3\sqrt{2}+3\sqrt{2}i. \end{cases}$$

Then zw in polar coordinates is given by:

A. $12\left\{\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)\right\}$

B. $12\left\{\cos\left(\frac{\pi}{3}\cdot\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{3}\cdot\frac{\pi}{4}\right)\right\}$

C. $\frac{1}{3}\left\{\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)\right\}$

D. $\frac{1}{3}\left\{\cos\left(\frac{\pi}{4}-\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{4}-\frac{\pi}{3}\right)\right\}$

E. $\frac{1}{3}\left\{\cos\left(\frac{\pi}{3}/\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{3}/\frac{\pi}{4}\right)\right\}$

10. The modulus of the complex number $\dfrac{2+i}{1-i}$ is:

A. $\dfrac{\sqrt{5}}{2}$

B. $\sqrt{10}$

C. $\dfrac{\sqrt{10}}{2}$

D. $\sqrt{5}$

E. $\dfrac{\sqrt{3}}{2}$

11. Use power series to approximate $\displaystyle\int_0^{0.1} \frac{x}{1+x^3}\,dx$ with error smaller than 10^{-7}.

A. 0.005002

B. 0.005001

C. 0.005000

D. 0.004999

E. 0.004998

12. Find the volume of the solid generated by revolving about the y–axis the region bounded by $y = 0, y = \dfrac{1}{1+x^4}$, $x = 0$, and $x = 2$.

A. $\pi \ln 3$

B. $\pi \tan^{-1}(4)$

C. 2π

D. $\pi \tan^{-1}(2)$

E. $\pi \ln 5$

13. Find the arc length of the curve given by the parametric equations

$$\begin{cases} x &= \sin^2 t, \\ y &= \dfrac{\cos 2t}{2}, \end{cases} \qquad 0 \le t \le \frac{\pi}{2}.$$

A. $\dfrac{\pi}{2}$

B. $\dfrac{\sqrt{2}}{2}\pi$

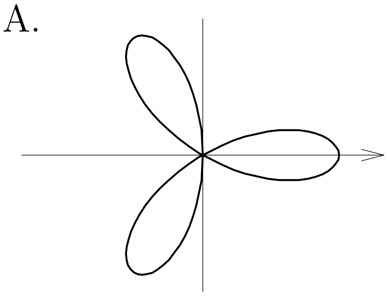
C. $\sqrt{2}\pi$

D. $\sqrt{2}$

E. 4

14. The graph of the polar equation $r = 2\cos 3\theta$ $(0 \le \theta \le \pi)$ looks most like

A.



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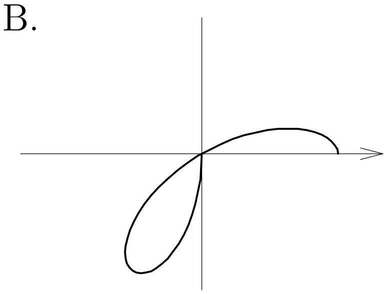

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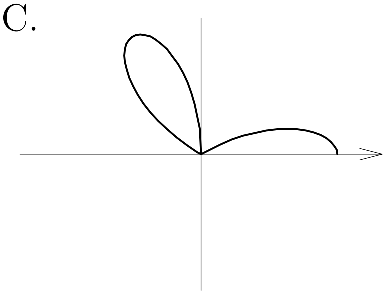

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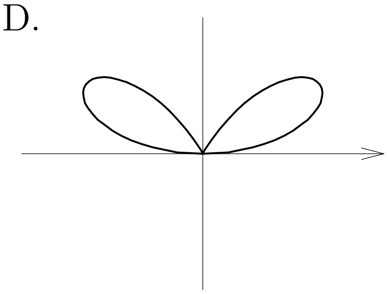

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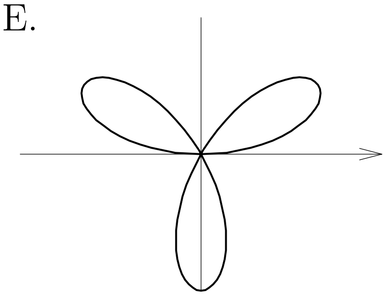

15. $\displaystyle\int_0^1 xe^x \, dx =$

A. 1

B. -1

C. e

D. $\dfrac{e^2}{2}$

E. $2e$

16. The lamina of uniform density bounded by the curves

$$y^2 = x, x = 4$$

has the center of mass

A. $(1, 0)$

B. $\left(\dfrac{12}{5}, 0\right)$

C. $\left(\dfrac{64}{5}, 0\right)$

D. $(3, 0)$

E. $(0, 0)$

17. We would like to compute the area of the surface generated by rotating the curve $y = e^x, 1 \leq x \leq 2$ about the x-axis.

Which of the following formulas compute(s) the area correctly ?

I. $\displaystyle\int_1^2 2\pi e^x \sqrt{1+e^{2x}}\, dx$ II. $\displaystyle\int_e^{e^2} 2\pi y \sqrt{1+\frac{1}{y^2}}\, dy$

III. $\displaystyle\int_1^2 2\pi x \sqrt{1+e^{2x}}\, dx$ IV. $\displaystyle\int_e^{e^2} 2\pi \ln(y) \sqrt{1+\frac{1}{y^2}}\, dy$

A. I only.

B. I and II only.

C. III and IV only.

D. III only.

E. All I, II, III, and IV.

18. Let $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{3^n}{(n+2)!}(x-1)^n$.

What is the 6-th derivative of f at $x = 1$?

A. $\dfrac{6!\ 3^6}{8!}$

B. 0

C. $\dfrac{3^6}{8!}(x-1)^6$

D. $\dfrac{3^6}{8!}$

E. $6!\ 3^6$

19. Consider the region bounded by the curves $y = x^2$ and $x = y^2$.

We want to compute the volume of the solid obtained by rotating the region about the line $x = 2$ by

(i) the Washer Method, and

(ii) the Cylindrical Shell method.

Choose the right formulas from below.

A. (i) $\displaystyle\int_0^1 \pi\{(\sqrt{y})^2 - (y^2)^2\}\,dy$

(ii) $\displaystyle\int_0^1 2\pi x\{\sqrt{x} - x^2\}\,dx$

B. (i) $\displaystyle\int_0^1 2\pi y\{\sqrt{y} - y^2\}\,dy$

(ii) $\displaystyle\int_0^1 \pi\{\sqrt{x} - x^2\}\,dx$

C. (i) $\displaystyle\int_0^1 \pi\{2 - (\sqrt{y} - y^2)\}^2\,dy$

(ii) $\displaystyle\int_0^1 2\pi(x+2)\{\sqrt{x} - x^2\}\,dx$

D. (i) $\displaystyle\int_0^1 \pi\{(2 - y^2)^2 - (2 - \sqrt{y})^2\}\,dy$

(ii) $\displaystyle\int_0^1 2\pi(2-x)\{\sqrt{x} - x^2\}\,dx$

E. (i) $\displaystyle\int_0^1 \pi\{(2 - \sqrt{y})^2 - (2 - y^2)^2\}\,dy$

(ii) $\displaystyle\int_0^1 2\pi(x-2)\{\sqrt{x} - x^2\}\,dx$

20. Which of the following series converges conditionally?

A. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

B. $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{(-1)^n}{n}\right)$

C. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+n}}$

D. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{e^n}{\sqrt{n}}$

E. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

21. Determine the interval of convergence for the series $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{\sqrt{n}} x^n$.

A. The series converges only for $x = 0$.

B. $\left(\frac{-1}{2}, \frac{1}{2}\right)$

C. $\left[\frac{-1}{2}, \frac{1}{2}\right)$

D. $\left[\frac{-1}{2}, \frac{1}{2}\right]$

E. $(-\infty, \infty)$.

22. Which of the following power series represents xe^{-x^2}?

A. $-\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$

B. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

C. $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$

D. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!}$

E. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n^2+1}}{(n^2+1)!}$

23. The first nonzero four terms of the Taylor series of $f(x) = \sin x$ at $x = \frac{\pi}{6}$ are

A. $\left(x - \frac{\pi}{6}\right) - \frac{\left(x - \frac{\pi}{6}\right)^3}{3!} + \frac{\left(x - \frac{\pi}{6}\right)^5}{5!} - \frac{\left(x - \frac{\pi}{6}\right)^7}{7!}$

B. $\frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) + \frac{1}{4}\left(x - \frac{\pi}{6}\right)^2 + \frac{\sqrt{3}}{12}\left(x - \frac{\pi}{6}\right)^3$

C. $\frac{1}{2}\left(x - \frac{\pi}{6}\right) + \frac{\sqrt{3}}{4}\left(x - \frac{\pi}{6}\right)^2 - \frac{1}{12}\left(x - \frac{\pi}{6}\right)^3 - \frac{\sqrt{3}}{48}\left(x - \frac{\pi}{6}\right)^4$

D. $\frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) - \frac{1}{4}\left(x - \frac{\pi}{6}\right)^2 - \frac{\sqrt{3}}{12}\left(x - \frac{\pi}{6}\right)^3$

E. $\frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) - \frac{1}{2}\left(x - \frac{\pi}{6}\right)^2 - \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)^3$

24. The series $\displaystyle\sum_{n=1}^{\infty}\left(\sin\left(\frac{1}{n^2}\right)\right)\left(e^{1/n}\right)$

A. converges by the Root Test.

B. diverges by the Comparison Test to the harmonic series.

C. diverges by the Test for Divergence.

D. converges by the Ratio Test.

E. converges by the Limit Comparison Test to $\displaystyle\sum \frac{1}{n^2}$.

25. The appropriate trigonometric substitution will convert the following integral

$$\int_5^7 \sqrt{x^2-6x+5}\;dx$$

into:

A. $\displaystyle 4\int_{\pi/4}^{\pi/3} \cos\theta\sin\theta\;d\theta$

B. $\displaystyle \int_0^{\pi/3} \sec\theta\tan^2\theta\;d\theta$

C. $\displaystyle 4\int_0^{\pi/3} \sec\theta\tan^2\theta\;d\theta$

D. $\displaystyle 4\int_{\tan^{-1}(2)}^{\tan^{-1}(4)} \tan\theta\sec^2\theta\;d\theta$

E. $\displaystyle \int_0^{\pi/2} \sin\theta\cos^2\theta\;d\theta$