MA166 — FINAL EXAM III — SPRING 2019 — MAY 03, 2019
TEST NUMBER 11

INSTRUCTIONS:

1. **Do not open the exam booklet until you are instructed to do so.**
2. **Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
3. **MARK YOUR TEST NUMBER ON YOUR SCANTRON**
4. Once you are allowed to open the exam, make sure you have a complete test. There are 15 different test pages (including this cover page).
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers on this test booklet.
6. The exam has 25 problems and each one is worth 8 points. The maximum possible score is 200 points. No partial credit.
7. Do not leave the exam room during the first 20 minutes of the exam.
8. If you do not finish your exam in the first 100 minutes, you must wait until the end of the exam period to leave the room.
9. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

DON'T BE A CHEATER:

1. Do not give, seek or obtain any kind of help from anyone to answer questions on this exam. If you have doubts, consult only your instructor.
2. Do not look at the exam or scantron of another student.
3. Do not allow other students to look at your exam or your scantron.
4. You may not compare answers with anyone else or consult another student until after you have finished your exam, given it to your instructor and left the room.
5. Do not consult notes or books.
6. **Do not handle** phones or cameras, calculators or any electronic device until after you have finished your exam, given it to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

STUDENT ID NUMBER: ______________________________

SECTION NUMBER AND RECITATION INSTRUCTOR: ______________________________

USEFUL FORMULAS

Trig Formulas:

$$\sin^2 x = \frac{1-\cos(2x)}{2}, \qquad \cos^2 x = \frac{1+\cos(2x)}{2}, \qquad \sec^2 x = 1+\tan^2 x$$

Useful Integrals:

$$\int \sec x\, dx = \ln|\sec x + \tan x| + C \quad \text{and} \quad \int \sqrt{1+x^2}\, dx = \frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\ln(x+\sqrt{1+x^2}) + C$$

Center of Mass:

$$\overline{x} = \frac{1}{A}\int_a^b x(f(x)-g(x))\, dx \qquad \text{and} \qquad \overline{y} = \frac{1}{A}\int_a^b \frac{1}{2}[(f(x))^2-(g(x))^2]\, dx$$

Arc Length, Surface Area and Volume:

Arc Length: $L = \int_a^b \sqrt{1+(f'(x))^2}\, dx$

Surface area: $S = 2\pi \int_a^b f(x)\sqrt{1+(f'(x))^2}\, dx$ or $S = 2\pi \int_a^b x\sqrt{1+(f'(x))^2}\, dx$

Volume by the washer method: $V = \pi \int_a^b (R^2(x) - r^2(x))\, dx$; $R(x)$ and $r(x)$ are the longer and shorter radii of the washer

Volume by cylindrical shells: $V = 2\pi \int_a^b x f(x)\, dx$

Maclaurin Series:

The geometric series: $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, provided $|x| < 1$

Logarithm: $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1}\frac{x^n}{n}$, provided $|x| < 1$

The exponential function: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for all x

Sine: $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ for all x

Cosine: $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ for all x

1. Find the angle (in radians) between the vectors $\mathbf{u} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$.

A. $\cos^{-1}(\frac{1}{6})$

B. $\cos^{-1}(\frac{1}{3})$

C. $\cos^{-1}(-\frac{1}{3})$

D. $\cos^{-1}(-\frac{1}{6})$

E. $\cos^{-1}(\frac{2}{3})$

2. Find the length of the projection $\mathbf{u} = \mathbf{i} + \mathbf{j} - \mathbf{k}$ onto $\mathbf{v} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$.

A. $\frac{1}{2}$

B. $\frac{2}{9}$

C. $\frac{2}{3}$

D. $\frac{1}{9}$

E. $\frac{1}{3}$

3. Find the area of the region enclosed by the curves $y = x^2 - 1$ and $y = -x^2 + 7$.

A. $\dfrac{16}{3}$

B. $\dfrac{64}{3}$

C. $\dfrac{32}{3}$

D. $\dfrac{32}{5}$

E. $\dfrac{64}{7}$

4. Find the volume of the solid obtained by rotating the region bounded by $y = 2x$ and $y = x^2$ about the x-axis.

A. $\dfrac{32}{15}\pi$

B. $\dfrac{2}{5}\pi$

C. $\dfrac{64}{15}\pi$

D. $\dfrac{3}{5}\pi$

E. $\dfrac{16}{15}\pi$

5. Find the volume of the solid obtained by rotating the region of the first quadrant bounded by $y = 1 + x^2$, $x = 0$, $x = 1$ and $y = 0$ about the y-axis.

A. π

B. 2π

C. $\dfrac{3\pi}{2}$

D. $\dfrac{\pi}{4}$

E. $\dfrac{2\pi}{3}$

6. Find the length of the curve $y = \dfrac{2}{3}x^{\frac{3}{2}}$ for $0 \leq x \leq 1$.

A. $\dfrac{2}{3}(2\sqrt{2} - 1)$

B. $\dfrac{2}{3}(2\sqrt{2} + 1)$

C. $\dfrac{3}{4}(\sqrt{3} - 1)$

D. $\dfrac{2}{3}(\sqrt{2} - 1)$

E. $\dfrac{2}{3}(2\sqrt{3} - 1)$

7. Find the area of the surface obtained by rotating $y = 1 + \frac{1}{2}x^2$ for $0 \le x \le 1$ about the y-axis

A. $\frac{2\pi}{3}(2\sqrt{3} - 1)$

B. $\frac{2\pi}{3}(\sqrt{5} - 1)$

C. $\frac{2\pi}{3}(\sqrt{3} - 1)$

D. $\frac{2\pi}{3}(\sqrt{2} - 1)$

E. $\frac{2\pi}{3}(2\sqrt{2} - 1)$

8. Let T be the trapezoid in the first quadrant bounded below by $y = 0$ and $0 \le x \le 2$, and on the left by $x = 0$ and $0 \le y \le 1$. For $0 \le x \le 1$, T is bounded from above by $y = 1$, and for $1 \le x \le 2$, T is bounded from above by the line $x + y = 2$. Find the x-coordinate of its center of mass.

A. $\frac{3}{4}$

B. $\frac{8}{9}$

C. $\frac{7}{9}$

D. $\frac{7}{3}$

E. $\frac{9}{8}$

9. Evaluate the integral $\int_0^{\frac{\pi}{2}} (\cos x)^3 \; dx$

A. $\frac{3}{4}$

B. $\frac{2}{3}$

C. $\frac{1}{5}$

D. $\frac{3}{2}$

E. $\frac{5}{8}$

10. Evaluate the integral $\int_0^{\frac{\pi}{3}} (\tan x)^3 (\sec x)^3 \; dx$

A. $\frac{58}{15}$

B. $\frac{48}{15}$

C. $\frac{64}{15}$

D. $\frac{32}{15}$

E. $\frac{82}{15}$

11. Evaluate the integral $\int_0^{\frac{\pi}{2}} x\cos(2x)dx$.

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. $\frac{1}{4}$

D. $-\frac{1}{4}$

E. $\frac{1}{3}$

12. A force of 5 lb. is required to stretch a spring $\frac{1}{3}$ ft. beyond its natural length. How much work is required to stretch the spring 2ft. beyond its natural length?

A. 10 ft-lbs.

B. 15 ft-lbs.

C. 20 ft-lbs.

D. 25 ft-lbs.

E. 30 ft-lbs.

13. Evaluate the integral $\displaystyle\int_0^1 \frac{1}{x^2+4x+3}\,dx$.

A. $\frac{1}{2}\ln 3$

B. $\frac{1}{2}\ln(\frac{3}{4})$

C. $\frac{1}{2}\ln(\frac{2}{3})$

D. $\frac{1}{2}\ln(\frac{8}{3})$

E. $\frac{1}{2}\ln(\frac{3}{2})$

14. Evaluate the integral $\displaystyle\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{dx}{(4x^2+1)^{\frac{3}{2}}}$.

A. $\frac{1}{4}(\sqrt{3}-\sqrt{2})$

B. $\frac{1}{4}(\sqrt{3}+\sqrt{2})$

C. $\frac{1}{4}(2-\sqrt{3})$

D. $\frac{1}{4}(\sqrt{3}-1)$

E. $\frac{1}{4}(\sqrt{2}-1)$

15. If $\dfrac{1}{x(x^2+1)} = \dfrac{A}{x} + \dfrac{Bx+C}{1+x^2}$, then $A+B+C$ is equal to:

A. 2

B. 1

C. $\dfrac{3}{2}$

D. 2

E. 0

16. Evaluate the improper integral $\displaystyle\int_0^\infty xe^{-x}dx$.

A. 1

B. $\dfrac{1}{4}$

C. $\dfrac{1}{2}$

D. $\dfrac{1}{3}$

E. The integral is divergent.

17. Compute the limit $\lim_{n\to\infty}\left(\sqrt{n^4+n^3+n^2}-\sqrt{n^4+n^3+2n^2+1}\right)$.

A. 0

B. 1

C. -1

D. $\frac{1}{2}$

E. $-\frac{1}{2}$

18. Find all values of p for which $\sum_{n=1}^{\infty}\frac{1}{(n^3+n)^p}$ converges.

A. $p > 1/3$

B. $p > 1$

C. $p \geq 1/3$

D. $p \geq 1$

E. $p < 1/3$

19. Which of the following series converge conditionally?

(i) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\sqrt{n}}$

(ii) $\sum_{n=2}^{\infty}(-1)^{n}\frac{1}{n(\ln(n))^2}$

(iii) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{2^n}$

A. (i) and (ii).

B. (i) only.

C. (i) and (iii).

D. (ii) and (iii).

E. (i), (ii), and (iii).

20. Find the interval of convergence for the power series $\sum_{n=1}^{\infty}\frac{n}{3^n}(x-1)^n$.

A. $-2 \leq x < 4$

B. $-2 < x \leq 4$

C. $-1 \leq x < 3$

D. $-2 < x < 4$

E. $-1 < x < 3$

21. Use one the formulas given on page 2 of the exam and the alternating series estimation theorem to compute $\int_0^{0.1} \ln(1+x)\,dx$ with an error less than 10^{-4}.

A. $\dfrac{1}{200} - \dfrac{1}{6000}$

B. $\dfrac{1}{300} - \dfrac{1}{6000}$

C. $\dfrac{1}{100} - \dfrac{1}{2000}$

D. $\dfrac{1}{500} - \dfrac{1}{5000}$

E. $\dfrac{1}{200} - \dfrac{1}{8000}$

22. Let $f(x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{(x-3)^{2n}}{6n^2}$. Find $f^{(10)}(3)$, the tenth derivative of f at 3.

A. $f^{(10)}(3) = (10)!$

B. $f^{(10)}(3) = -(10)!$

C. $f^{(10)}(3) = -\dfrac{(10)!}{150}$

D. $f^{(10)}(3) = \dfrac{(10)!}{250}$

E. $f^{(10)}(3) = \dfrac{(10)!}{500}$

23. The Taylor series of the function $f(x) = \dfrac{1}{x+5}$ centered at 2 is equal to

A. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{5^{n+1}}(x-2)^n$

B. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}}(x-2)^n$

C. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{5^{n+1}}(x-2)^n$

D. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{7^{n+1}}(x-2)^n$

E. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{7^{n+1}}(x-2)^n$

24. Find the slope of the tangent line at the point $(2, 0)$, for the curve parameterized by $x = 2t^2, y = t^3 - t$, $t > 0$.

A. $-\dfrac{1}{3}$

B. $-\dfrac{1}{2}$

C. $\dfrac{2}{3}$

D. $\dfrac{1}{3}$

E. $\dfrac{1}{2}$

25. Let $z = 1 + \sqrt{3}\,\mathrm{i}$. Find z^6.

A. $z^6 = 64$

B. $z^6 = -64$

C. $z^6 = 64(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\mathrm{i})$

D. $z^6 = 64(\frac{1}{2} + \frac{\sqrt{3}}{2}\,\mathrm{i})$

E. $z^6 = 64(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\mathrm{i})$