1. Evaluate \( \int x \sin(4x) \, dx \).

\[ \int x \sin(4x) \, dx = \ldots \]

2. Find the equation of the sphere passing through \( P(-6, 5, 2) \) and \( Q(2, -3, 3) \) with its center at the midpoint of \( PQ \).

The standard equation of the sphere is \( \ldots \).

3. Integrate the given function.

\[ \int 4 \cos^2 x \, dx \]

\[ \int 4 \cos^2 x \, dx = \ldots \]

4. Evaluate the integral or state that it diverges.

\[ \int_3^\infty \frac{x}{(x + 4)^2} \, dx \]

Select the correct choice and, if necessary, fill in the answer box to complete your choice.

- \( \text{A.} \) The integral converges to \( \ldots \).
- \( \text{B.} \) The integral diverges.

5. Find the area of the parallelogram that has adjacent sides \( \mathbf{u} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k} \) and \( \mathbf{v} = 2\mathbf{j} - \mathbf{k} \).

The area of the parallelogram is \( \ldots \).

(Type an exact answer, using radicals as needed.)
6. Evaluate the following integral using trigonometric substitution.

\[ \frac{9 \sqrt{2}}{2} \int_{0}^{9} \frac{x^2}{\sqrt{81 - x^2}} \, dx \]

What substitution will be the most helpful for evaluating this integral?

- A. \( x = 9 \sin \theta \)
- B. \( x = 9 \sec \theta \)
- C. \( x = 9 \tan \theta \)

Rewrite the given integral using this substitution.

\[ \frac{9 \sqrt{2}}{2} \int_{0}^{9} \frac{x^2}{\sqrt{81 - x^2}} \, dx = \int_{0}^{\theta} \left( \text{expression} \right) \, d\theta \]

(Simplify your answers. Type exact answers.)

Evaluate the integral.

\[ \frac{9 \sqrt{2}}{2} \int_{0}^{9} \frac{x^2}{\sqrt{81 - x^2}} \, dx = \text{value} \] (Type an exact answer.)

7. Evaluate \( \int \cos^3 \left( \frac{x}{18} \right) \, dx \).

\[ \int \cos^3 \left( \frac{x}{18} \right) \, dx = \text{value} \]
8. Determine whether the following series converges.

\[ \sum_{k=1}^{\infty} \frac{8}{2 \ln k + 2} \]

Select the correct choice below and, if necessary, fill in the answer box to complete your choice. (Type an exact answer.)

- **A.** Since \[ \int_{1}^{\infty} \frac{8}{2 \ln \xi + 2} \, d\xi = \quad \text{, the series converges by the Integral Test.} \]
- **B.** The terms of the series are alternating and their limit is \( \quad \text{, so the series converges by the Alternating Series Test.} \)
- **C.** The Ratio Test yields \( r = \quad \text{. This is less than 1, so the series converges by the Ratio Test.} \)
- **D.** Because \( \frac{8}{k + 2} \leq \frac{8}{2 \ln k + 2} \), for any positive integer \( k \), and \( \sum_{k=1}^{\infty} \frac{8}{k + 2} \) diverges, the series

- **E.** Since \[ \int_{1}^{\infty} \frac{8}{2 \ln \xi + 2} \, d\xi = \infty \text{, the series diverges by the Integral Test.} \]
- **F.** Because \( \frac{8}{k + 2} \geq \frac{8}{2 \ln k + 2} \), for any positive integer \( k \), and \( \sum_{k=1}^{\infty} \frac{8}{k + 2} \) converges, the series

converges by the Comparison Test.

9. For the vectors \( \mathbf{u} = \langle -4, 0, 4 \rangle \) and \( \mathbf{v} = \langle 1, 3, -3 \rangle \), calculate \( \text{proj}_v \mathbf{u} \) and \( \text{scal}_v \mathbf{u} \).

\[ \text{proj}_v \mathbf{u} = \langle \quad , \quad , \quad \rangle \]

\[ \text{scal}_v \mathbf{u} = \quad \]

(Type an exact answer, using radicals as needed.)

10. Make a sketch of the region and its bounding curves. Find the area of the region.

The region inside one leaf of \( r = 4 \cos 5\theta \)

Choose the correct graph of the region below.

- **A.**
- **B.**
- **C.**
- **D.**

The area of the region is \( \quad \) square units. (Type an exact answer, using \( \pi \) as needed.)
11. Find the slope of the line tangent to the polar curve at the given point.

\[ r = 8 \sin \theta; \left( 4, \frac{\pi}{6} \right) \]

Find \( \frac{dy}{dx} \) as a function of \( \theta \).

\[ \frac{dy}{dx} = \] __________

Select the correct choice below and, if necessary, fill in the answer box within your choice.

- A. The slope of the curve at the point \( \left( 4, \frac{\pi}{6} \right) \) is __________. (Type an exact answer.)
- B. The slope of the curve at the point \( \left( 4, \frac{\pi}{6} \right) \) is undefined.

12. Evaluate the following integral using trigonometric substitution.

\[ \int \frac{\sqrt{196 - x^2}}{x} \, dx \]

What substitution will be the most helpful for evaluating this integral?

- A. \( x = 14 \sec \theta \)
- B. \( x = 14 \sin \theta \)
- C. \( x = 14 \tan \theta \)

Rewrite the given integral using this substitution.

\[ \int \frac{\sqrt{196 - x^2}}{x} \, dx = \int (\text{___________}) \, d\theta \]

(Simplify your answers. Type exact answers.)

Evaluate the integral.

\[ \int \frac{\sqrt{196 - x^2}}{x} \, dx = \] __________

(Type an exact answer.)
13. Use the Integral Test to determine the convergence or divergence of the following series, or state that the test does not apply.

\[ \sum_{k=1}^{\infty} k \cdot e^{-6k^2} \]

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The series converges. The value of the integral \( \int_{1}^{\infty} x \cdot e^{-6x^2} \, dx \) is ________.
- **B.** The series diverges. The value of the integral \( \int_{1}^{\infty} x \cdot e^{-6x^2} \, dx \) is ________.
- **C.** The Integral Test does not apply.

14. Find the area of the surface generated when the given curve is revolved about the x-axis.

\[ y = \frac{x^3}{3} + \frac{1}{4x}, \text{ for } \frac{1}{2} \leq x \leq 1 \]

The area of the surface is ________ square units. (Type an exact answer, using \( \pi \) as needed.)

15. Convert the equation \( r = -6 \cos \theta \) to Cartesian coordinates. Describe the resulting curve.

Choose the correct equation below.

- **A.** \( x^2 + (y + 3)^2 = 9 \)
- **B.** \( (x + 3)^2 + y^2 = 9 \)
- **C.** \( x^2 + (y - 3)^2 = 9 \)
- **D.** \( (x - 3)^2 + y^2 = 9 \)

The equation describes a circle.

The center of the circle is ________. (Simplify your answer. Type an ordered pair.)

The radius is ________. (Simplify your answer.)

16. Find the length of the cardioid \( r = 1 + \cos \theta \).

The length of the cardioid is ________.

(Simplify your answer.)
17. Use the Divergence Test to determine whether the following series diverges or state that the test is inconclusive.

\[ \sum_{n=0}^{\infty} \frac{n}{7n+1} \]

Select the correct answer below and fill in the answer box to complete your choice.

- **A.** According to the Divergence Test, the series converges because \( \lim_{k \to \infty} a_k = \) (Simplify your answer.)
- **B.** According to the Divergence Test, the series diverges because \( \lim_{k \to \infty} a_k = \) (Simplify your answer.)
- **C.** The Divergence Test is inconclusive because \( \lim_{k \to \infty} a_k = \) (Simplify your answer.)
- **D.** The Divergence Test is inconclusive because \( a_k \) does not exist.

18. Use the general slicing method to find the volume of the following solid.

The solid with a semicircular base of radius 19 whose cross sections perpendicular to the base and parallel to the diameter are squares

The volume of the solid is \( \) cubic units.

19. Determine whether the following series converges.

\[ \sum_{k=1}^{\infty} (-1)^k \sin \frac{1}{k} \]

Let \( a_k \geq 0 \) represent the magnitude of the terms of the given series. Select the correct choice below and fill in the answer box(es) to complete your choice.

- **A.** The series converges because \( a_k = \) is nonincreasing in magnitude for \( k \) greater than some index \( N \).
- **B.** The series diverges because \( a_k = \) is nondecreasing in magnitude for \( k \) greater than some index \( N \) and \( \lim_{k \to \infty} a_k = \).
- **C.** The series converges because \( a_k = \) is nondecreasing in magnitude for \( k \) greater than some index \( N \) and \( \lim_{k \to \infty} a_k = \).
- **D.** The series converges because \( a_k = \) and for any index \( N \), there are some values of \( k > N \) for which \( a_{k+1} \geq a_k \) and some values of \( k > N \) for which \( a_{k+1} \leq a_k \).
- **E.** The series diverges because \( a_k = \) is nonincreasing in magnitude for \( k \) greater than some index \( N \).
- **F.** The series diverges because \( a_k = \) and for any index \( N \), there are some values of \( k > N \) for which \( a_{k+1} \geq a_k \) and some values of \( k > N \) for which \( a_{k+1} \leq a_k \).
20. Convert the following equation to Cartesian coordinates. Describe the resulting curve.

\[ r = \frac{6}{5 \cos \theta + 7 \sin \theta} \]

Write the Cartesian equation.

_____________________________

Describe the curve. Select the correct choice below and, if necessary, fill in any answer box(es) to complete your choice.

- **A.** The curve is a line with slope  and a y-intercept at the point  .
  (Type exact answers, using radicals as needed.)

- **B.** The curve is a vertical line with x-intercept at the point  .
  (Type exact answers, using radicals as needed.)

- **C.** The curve is a circle centered at the point  with radius  .
  (Type exact answers, using radicals as needed.)

21. A 60-m-long chain hangs vertically from a cylinder attached to a winch. Assume there is no friction in the system and that the chain has a density of 5 kg/m. Use 9.8 m/s² for the acceleration due to gravity.

   a. How much work is required to wind the entire chain onto the cylinder using the winch?
   b. How much work is required to wind the chain onto the cylinder if a 35-kg block is attached to the end of the chain?

   **a.** Set up the integral that gives the work required to wind the entire chain onto the cylinder using the winch. Use increasing limits of integration.

   \[ \int (\text{___________}) \, dy \]

   (Type exact answers.)

   The amount of work required is  (1)  .
   (Type an integer or a decimal.)

   **b.** Set up the integral that gives the work required to wind the chain onto the cylinder if a 35-kg block is attached to the end of the chain. Use increasing limits of integration.

   \[ \int (\text{___________}) \, dy \]

   (Type exact answers.)

   The amount of work required if a 35-kg block is attached to the end of the chain is  (2)  .
   (Type an integer or a decimal.)
22. Evaluate the following definite integral.

\[ \int_{-1}^{9} \frac{12y}{y^2 - 8y - 20} \, dy \]

Find the partial fraction decomposition of the integrand.

\[ \int_{-1}^{9} \frac{12y}{y^2 - 8y - 20} \, dy = \int_{-1}^{9} \left( \frac{\text{__________}}{y^2 - 8y - 20} \right) \, dy \]

Evaluate the definite integral.

\[ \int_{-1}^{9} \frac{12y}{y^2 - 8y - 20} \, dy = \text{__________} \]

(Type an exact answer.)

23. Let R be the region bounded by the following curves. Use the shell method to find the volume of the solid generated when R is revolved about the x-axis.

\[ y = \sqrt{x}, \ y = 0, \text{ and } x = 121 \]

Set up the integral that gives the volume of the solid. Use increasing limits of integration. Select the correct choice below and fill in the answer boxes to complete your choice.

(Type an exact answer.)

\[ \text{A. } \int \text{__________} \, dy \]

\[ \text{B. } \int \text{__________} \, dx \]

The volume is _________. (Type an exact answer.)
24. Replace \( x \) by \( \frac{x}{5} - 1 \) in the series 
\[
\ln (1 + x)^6 = 6 \sum_{k=1}^{\infty} \frac{(-1)^{k+1}x^k}{k}
\]
to obtain a power series for \( \ln \left( \frac{x}{5} \right)^6 \) centered at \( x = 5 \).

What is the interval of convergence for the new power series?

Choose the correct power series below.

- **A.** \( 6 \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(x-5)^k}{5^k k} \)
- **B.** \( 6 \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(x-1)^k}{k} \)
- **C.** \( 6 \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(x-5)^k}{k} \)
- **D.** \( 6 \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(x-1)^k}{5^k k} \)

The interval of convergence is ___________________.
(Simplify your answer. Type your answer in interval notation.)

25. Let \( R \) be the region bounded by the following curves. Find the volume of the solid generated when \( R \) is revolved about the y-axis.

\[
y = \sqrt{x}, \quad y = 0, \quad x = 9
\]

Set up the integral that gives the volume of the solid.

\[
\int_{0}^{9} (\_\_\_\_\_) \, dy
\]
(Type exact answers.)

The volume of the solid is __________ cubic units.
(Type an exact answer.)