NAME________________________________INSTRUCTOR________________________________

1. You must use a #2 pencil on the scantron answer sheet.
2. Fill in your name, your four digit section number, and your student identification number. If you do not know your section number, ask your instructor. (Leave the test/quiz number blank.) Sign your name.
3. There are 15 questions. Blacken in your choice of the correct answer in the spaces provided on the scantron answer sheet. **Only the scantron answer sheet will be graded**.
4. The exam is self-explanatory. Do not ask your instructor any questions about the exam problems.
5. Only one-line calculators (any brand) are allowed. Cell phones and PDA's may not be used as a calculator and must be put away during the exam. NO BOOKS OR PAPERS ARE ALLOWED.

LAGRANGE EQUATIONS

For the function $f(x,y)$ subject to the constraint $g(x,y)=k$, the Lagrange equations are

$$f_x = \lambda g_x \qquad f_y = \lambda g_y \qquad g(x,y) = k$$

GEOMETRIC SERIES

If $0 < |r| < 1$, then

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

TAYLOR SERIES

The Taylor series of $f(x)$ about $x=a$ is the power series

$$\sum_{n=0}^{\infty} a_n (x-a)^n \quad \text{where} \quad a_n = \frac{f^{(n)}(a)}{n!}$$

Examples:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } -\infty < x < \infty; \qquad \ln x = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}(x-1)^n, \text{ for } 0 < x \le 2$$

VOLUME & SURFACE AREA

Right Circular Cylinder	**Sphere**	**Right Circular Cone**
$V = \pi r^2 h$	$V = \frac{4}{3}\pi r^3$	$V = \frac{1}{3}\pi r^2 h$
$SA = \begin{cases} 2\pi r^2 + 2\pi r h \\ \pi r^2 + 2\pi r h \end{cases}$	$SA = 4\pi r^2$	$SA = \pi r\sqrt{r^2+h^2} + \pi r^2$

1. Evaluate

$$\int_0^1 \int_0^{\sqrt{2}} 2xy\,dx\,dy.$$

A) $\sqrt{2}$

B) 1

C) $\frac{\sqrt{2}}{2}$

D) $2\sqrt{2}$

E) 4

2. Determine whether or not the series below converges or diverges. If it converges, find its sum.

$$\sum_{n=0}^{\infty} \frac{3^{n-1}}{7^{n+1}}$$

A) Converges to 0

B) Converges to $\frac{1}{12}$

C) Converges to $\frac{147}{4}$

D) Diverges

E) Converges to $\frac{7}{4}$

3. Use the ratio test to determine the interval of absolute convergence of the following power series:

$$\sum_{k=0}^{\infty} \frac{x^k}{3^k}$$

A) $-\sqrt{3} < x < \sqrt{3}$

B) $-9 < x < 9$

C) $-3 < x < 3$

D) $-1/3 < x < 1/3$

E) $-1/\sqrt{3} < x < 1/\sqrt{3}$

4. In Purdue's Chemistry department, the chemists have found that in a water based solution containing 10 grams of certain undissolved chemicals, the rate of change of the amount of chemicals dissolved in the solution is proportional to the amount of the undissolved chemicals. Let $Q(t)$ (in grams) be the amount of dissolved chemicals at time t, k is the *positive* proportionality constant, then the differential equation describing the given situation is:

   A) $\dfrac{dQ}{dt} = -kQ$

   B) $\dfrac{dQ}{dt} = k(10 - Q)$

   C) $\dfrac{dQ}{dt} = k\dfrac{Q}{10}$

   D) $\dfrac{dQ}{dt} = k\dfrac{10}{Q}$

   E) $\dfrac{dQ}{dt} = kQ$

5. It is John's birthday and his parents want to make him a cake in the shape of a rectangular box. The height of the cake will be 30 centimeters, and three times the width plus two times the length will be 240 centimeters. Find the largest possible volume of cake that John can receive.

   A) 0.036 cubic meters

   B) 0.048 cubic meters

   C) 0.052 cubic meters

   D) 0.069 cubic meters

   E) 0.072 cubic meters

6. Evaluate

$$\int_0^3 \int_0^{\sqrt{y}} x e^{y^2}\, dx\, dy.$$

A) $\frac{9}{2}e^9$

B) $\frac{9}{4}e^9$

C) $\frac{1}{2}\left(e^9 - 1\right)$

D) $\frac{1}{4}\left(e^9 - 1\right)$

E) e^9

7. Use summation notation to write the given series in compact form, starting the series at $n = 1$.

$\dfrac{5}{2} - \dfrac{7}{4} + \dfrac{9}{8} - \dfrac{11}{16} + \dots$

A) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{2n+5}{2^{n+1}}$

B) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{2n+3}{2^n}$

C) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n+3}{2^n}$

D) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{2^{n-1}}$

E) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n+5}{n^2}$

8. A ball has the property that each time it falls from a height h onto the ground, it will rebound to a height of rh, where r $(0 < r < 1)$ is called the coefficient of resititution. Find the total distance traveled by a ball with $r = 0.5$ that is dropped from a height of 5 meters.

   A) 30

   B) 10

   C) 20

   D) 15

   E) 25

9. Determine the first 2 non-zero terms of the Taylor series of $f(x) = (1 + x)\,e^x$, about $x = -1$.

   A) $\dfrac{2!}{e}(x+1) + \dfrac{3!}{e}(x+1)^2$

   B) $2(x+1) + 3(x+1)^2$

   C) $\dfrac{1}{2!}(x+1) + \dfrac{1}{3!}(x+1)^2$

   D) $e^2(x+1) + e^3(x+1)^2$

   E) $\dfrac{1}{e}(x+1) + \dfrac{1}{e}(x+1)^2$

10. Find the general solution of the following differential equation:

$$\frac{dy}{dt} = e^{-t-y}$$

A) $y = -e^{-t} + C$

B) $y = -e^{-t-y}$

C) $y = \ln(C - e^{-t})$

D) $y = -\ln(e^{-t} + C)$

E) $y = \ln(e^{-t} + C)$

11. Evaluate

$$\iint\limits_R (6xy + 12)\, dA,$$

where R is the region bounded by $y = x^2$ and $y = x + 2$.

A) 87.75

B) 47.5

C) 20.25

D) 13.5

E) 62.75

12. Determine which of the following series definitely converge.

I. $\displaystyle\sum_{k=1}^{\infty} \frac{(-\pi)^{k+4}}{e^{2k}}$

II. $\displaystyle\sum_{k=2}^{\infty} \frac{1-k}{2k-2}$

III. $\displaystyle\sum_{k=3}^{\infty} \frac{(5k)^3}{(k-1)!}$

A) III

B) I & II

C) I & III

D) II & III

E) II

13. A jeweler is making gold and silver rings for his friends. He has estimated that if x grams of gold and y grams of silver are used, the number of rings he can make is

$$R(x,y) = 30x^{\frac{1}{2}}y^{\frac{3}{5}}.$$

If he is only planning to use 150 grams of metal, how many grams of gold should he use to maximize the number of rings?

A) 87.82

B) 30.32

C) 47.50

D) 68.18

E) 102.5

14. In an experiment, a scientist adds a batch of 1000 active bacteria to a sample every hour. The fraction of bacteria that remain active in the sample after t hours is $f(t) = e^{-0.3t}$. If the experiment continues indefinitely, approximately how many active bacteria will eventually be present in the sample, just after a batch of active bacteria is added?

A) 741

B) 2,860

C) 1,000

D) 3,858

E) 4,851

15. The Taylor series of a function $f(x)$ about $x = 0$ happens to be $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2 \cdot n!} x^{2n}$. What is $f(x)$?

A) e^{-x^2}

B) e^{x^2}

C) $\dfrac{e^{-x^2}}{2}$

D) $\dfrac{e^{x^2}}{2}$

E) $\dfrac{e^{-x^2} - 1}{2}$