MA 261 EXAM 1 Name____________________________________

1. Find the equation of the plane containing $(0, 1, 2)$ and whose normal is perpendicular to both $\bar{a} = \bar{i} + \bar{j}$, $\bar{b} = \bar{j} - \bar{k}$.

A. $x + y + z = 3$

B. $-x + y + z = 3$

C. $x - y - z = 3$

D. $x + y + z = -3$

E. None of the above

2. The distance between the plane
$$2x + y + 2z = 4$$
and the point $(1, 7, 2)$ is

A. 1

B. 2

C. 3

D. 4

E. None of the above

3. A unit tangent vector to the graph of $y = 2x^3$ at $(1, 2)$ is given by

A. $\dfrac{\bar{i} + 6\bar{j}}{\sqrt{37}}$

B. $\dfrac{\bar{i} + 4\bar{j}}{\sqrt{17}}$

C. $\dfrac{\bar{i} - \bar{j}}{\sqrt{2}}$

D. $\dfrac{2\bar{i} + 3\bar{j}}{\sqrt{13}}$

E. $\dfrac{\bar{i} + 2\bar{j}}{\sqrt{5}}$

4. A particle is moving with acceleration $4\bar{j} + 6t\bar{k}$. If the position at time $t = 1$ is $\bar{r}(1) = \bar{i} + 3\bar{j} + \bar{k}$ and the velocity at time $t = 0$ is $\bar{v}(0) = \bar{i} + \bar{j}$, then the position at time $t = 2$ is

A. $4\bar{i} + 10\bar{j} + 10\bar{k}$

B. $\bar{i} + 4\bar{j} + 10\bar{k}$

C. $\bar{i} + \frac{8}{3}\bar{j} + 4\bar{k}$

D. $2\bar{i} + 10\bar{j} + 8\bar{k}$

E. $2\bar{i} + 8\bar{j} + 8\bar{k}$

5. Which of the following surfaces represents the graph of $z = \frac{x^2}{4} + y^2$ in the 1st octant.

6. If $f(x,y) = \dfrac{3x^2 + yx}{x^2 + y^2}$, $(x,y) \neq (0,0)$, let ℓ be the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along the y-axis, and let m be the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along the line $y = x$. Then

A. $\ell = 3, \quad m = 2$

B. $\ell = 0, \quad m = 2$

C. $\ell = 0, \quad m = \frac{3}{2}$

D. $\ell = 3, \quad m = 3$

E. $\ell = \frac{1}{2}, \quad m = \frac{1}{2}$

7. Find a value of a for which the function $z = 4\cos(x + ay)$ satisfies $\dfrac{\partial^2 z}{\partial y^2} = 9\ \dfrac{\partial^2 z}{\partial x^2}$.

A. $a = 2$

B. $a = 0$

C. $a = \frac{1}{2}$

D. $a = 1$

E. $a = 3$

8. Find the maximal directional derivative of

$$f(x, y, z) = e^x + e^y + e^{2z}$$

at $(1, 1, -1)$.

A. $e\sqrt{3 - 2e}$

B. $\sqrt{2e^2 + 4e^{-4}}$

C. $\frac{1}{e}\sqrt{2 - 4e^{-3}}$

D. $\sqrt{2e^2 + e^{-4}}$

E. $\sqrt{e^2 + 2e^{-4}}$

9. Find symmetric equations of the line containing $(1, 2, 3)$ and perpendicular to the plane $2x + 3y - z = 8$.

10. Find the length of the curve
$\bar{r}(t) = \frac{t^2}{2}\,\bar{i} + 7\bar{j} + \frac{t^3}{3}\,\bar{k}$, $0 \le t \le 2$.

11. (a) Complete the following definition of f_y at $(0,0)$:

$$f_y(0,0) = \lim_{h \to 0}$$

(b) If $f(x,y) = \begin{cases} \frac{x+y^3}{3x^2+4y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$, compute $f_y(0,0)$ by evaluating the above limit.

$f_y(0,0) =$

12. A right circular cylinder has a radius and altitude that vary with time. At a certain instant the altitude is increasing at 0.5 ft/sec and the radius is decreasing at 0.2 ft/sec. How fast is the volume changing if at this time the radius is 20 feet and the altitude is 60 feet.