

EXAM 1

MATH 26100

FALL 2011

Name _____

Student ID _____

Recitation Instructor _____

Recitation Section and Time _____

INSTRUCTIONS:

1. This exam contains 11 problems each worth 9 points (one point free).
2. Please supply all information requested above on the scantron.
3. Work only in the space provided, or on the backside of the pages. You must show your work.
4. Mark your answers clearly on the scantron. Also circle your choice for each problem in this booklet.
5. No books, notes, or calculators, please.

Mark TEST 01 on your scantron!

Key

D E E C

B B E A D A C

1. Let C be the curve given by $\vec{r}(t) = \langle 4\sqrt{t}, t, 5 - t^2 \rangle$ for $t > 0$. At what point does the tangent line to C at $(4, 1, 4)$ intersect the xy plane?

- A. $(0, 1, 0)$
- B. $(4\sqrt{5}, \sqrt{5}, 0)$
- C. $(2, 1, 0)$
- D. $(8, 3, 0)$
- E. $(0, -1, 0)$

2. The arclength of the curve $\vec{r}(t) = 2t\vec{i} + t^2\vec{j} + (\ln t)\vec{k}$ for $2 \leq t \leq 4$ is:

- A. $\frac{17}{4}$
- B. $4 + \ln 2$
- C. $16 + \ln 2$
- D. $\frac{15}{4}$
- E. $12 + \ln 2$

3. A particle moves in space with acceleration $\vec{a}(t) = e^t \vec{k}$ and initial velocity and position given by $\vec{v}(0) = \vec{0}$, $\vec{r}(0) = \vec{j} + \vec{k}$. Where is the particle at time $t = 2$?

- A. $(1, 1, e^2)$
- B. $(0, 1, e^2)$
- C. $(0, 1, e - 1)$
- D. $(1, 1, e^2 - 2)$
- E. $(0, 1, e^2 - 2)$

4. Suppose that z is defined implicitly as a function of x and y by the equation

$$e^{yz} + \sin(\pi yz) - xyz = 0.$$

What is the value of $\frac{\partial z}{\partial x}$ at $(e, 1, 1)$?

- A. $-\frac{1}{e}$
- B. $\frac{1}{e}$
- C. $-\frac{1}{\pi}$
- D. $\frac{1}{\pi}$
- E. $\frac{1}{e - \pi}$

5. The surface area of a rectangular box is given by the function

$$S(x, y, z) = 2xy + 2yz + 2xz$$

where x, y, z are its sides. These are measured as $x = 10$ cm, $y = 20$ cm, $z = 30$ cm with possible errors in measurements as much as 0.1 cm. Use differentials to estimate the maximum error in the calculated surface area.

- A. 12 cm²
- B. 24 cm²
- C. 36 cm²
- D. 48 cm²
- E. 60 cm²

6. Given $\vec{a} = \langle 1, -1, 2 \rangle$ and $\vec{b} = \langle 2, 1, 0 \rangle$, find t such that the vector $\vec{c} = \langle 5, t - 1, 2 \rangle$ is perpendicular to $\vec{a} \times \vec{b}$.

A. $t = 1$

B. $t = 2$

C. $t = -1$

D. $t = -2$

E. $t = 0$

7. The intersection of the hyperbolic paraboloid $x^2 - y^2 - z - 1 = 0$ with the yz -plane consists of

A. a hyperbola and a parabola

B. a hyperbola

C. an ellipse

D. two lines

E. a parabola

8. Let $f(x, y) = \sqrt{x^2 + y}$. The equation for the tangent plane to $z = f(x, y)$ at $(2, 1)$ is

A. $2\sqrt{5}z - 4x - y = 1$

B. $2\sqrt{5}z - 4x - y = 10$

C. $2z - 2x - y = 1$

D. $2z - 2x - y = 10$

E. $2\sqrt{5}z - 2x - y = 9$

9. The critical points of $f(x, y) = 3x^3 + 3y^3 + x^3y^3$ are:

A. $(0, 0), (1, -1)$

B. $(0, 0)$

C. $(1, 1)$

D. $(0, 0), (-3^{1/3}, -3^{1/3})$

E. $(-3^{1/3}, -3^{1/3}), (1, 1)$

10. The directional derivative of the function $f(x, y) = 4xy + e^{xy}$ at the point $(0, 1)$ and in the direction of $\vec{v} = \langle 3, -4 \rangle$ is:

- A. 3
- B. 15
- C. $\langle 5, 0 \rangle$
- D. $\langle 3, -4 \rangle$
- E. -15

11. If

$$z = \frac{1}{u^2 + v}, \quad u(s, t) = t + s^2, \quad v(s, t) = \ln(t)$$

then $\frac{\partial z}{\partial t}$ is:

- A. $\frac{-(1 + \frac{1}{t})}{(t + s^2)^2 + \ln t}$
- B. $\frac{-(2(t + s^2) + \frac{\ln t}{t})}{(t + s^2)^2 + \ln t}$
- C. $\frac{-(2(t + s^2) + \frac{1}{t})}{((t + s^2)^2 + \ln t)^2}$
- D. $\frac{-(u + 1)}{(u^2 + v)^2}$
- E. $\frac{-(2u + 1)}{u^2 + v}$