MA 26100
EXAM 1 Green
February 21, 2018

NAME ______________________ YOUR TA'S NAME ______________________

STUDENT ID # ____________________ RECITATION TIME ____________________

Be sure the paper you are looking at right now is GREEN! Write the following in the TEST/QUIZ NUMBER boxes (and blacken in the appropriate spaces below the boxes): **00**

You must use a #2 pencil on the mark–sense sheet (answer sheet). On the mark–sense sheet, fill in your TA's name and the COURSE number. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. Fill in your four-digit SECTION NUMBER. If you do not know your section number, ask your TA. Sign the mark–sense sheet.

There are **12** questions, each worth 8 points (you will automatically earn 4 points for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–12. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the mark–sense sheet and the exam booklet when you are finished.

If you finish the exam before 8:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20. If you don't finish before 8:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ____

STUDENT SIGNATURE: ____

1. Identify the surface defined by $x^2 - y^2 - 4x + z^2 = 4$.

A. hyperboloid of one sheet

B. hyperbolic paraboloid

C. hyperboloid of two sheets

D. ellipsoid

E. cone

2. If L is the tangent line to the curve $\vec{\mathbf{r}}(t) = \langle 2t - 1,\ t^2,\ t^2 - 2 \rangle$ at $(3, 4, 2)$, find the point where L intersects the xy-plane.

A. $(2, 1, 0)$

B. $(1, 2, 0)$

C. $(2, -2, 0)$

D. $(2, 2, 0)$

E. $(0, 0, 0)$

3. Let $\vec{\mathbf{v}} = \displaystyle\int_0^1 \left(\frac{1}{2}\vec{\mathbf{i}} + 2t^3\vec{\mathbf{j}} + (t - 3t^2)\vec{\mathbf{k}}\right)\, dt$. Compute $|\vec{\mathbf{v}}|$.

A. 1

B. $\dfrac{3}{2}$

C. $\dfrac{1}{4}$

D. $\dfrac{1}{2}$

E. $\dfrac{\sqrt{3}}{2}$

4. Find the area of the triangle with vertices at $P(2, 2, 1)$, $Q(1, -1, 2)$, and $R(0, 1, -1)$.

A. $\sqrt{5}$

B. $\dfrac{3\sqrt{10}}{2}$

C. $\dfrac{\sqrt{31}}{2}$

D. $2\sqrt{5}$

E. $\dfrac{\sqrt{69}}{2}$

5. The level curves of $f(x, y) = \sqrt{x^2 + 4y^2 + 4} - x$ are

A. hyperbolas

B. ellipses

C. parabolas

D. sometimes lines and sometimes ellipses

E. circles

6. Find the length of the curve:

$$\vec{\mathbf{r}}(t) = \langle 4\sin t,\ 3t,\ -4\cos t\rangle, \quad 0 \le t \le \frac{1}{2}.$$

A. $\dfrac{8}{3}\sinh^{-1}\left(\dfrac{3}{8}\right)$

B. $\dfrac{8}{3}\sinh^{-1}\left(\dfrac{3}{8}\right) + \dfrac{\sqrt{73}}{8}$

C. 2.5

D. 5

E. 5π

7. A particle is moving with acceleration

$$\vec{\mathbf{a}}(t) = \langle 6,\ 6t,\ 0\rangle.$$

If at time $t = 1$, the particle has position $\vec{\mathbf{r}}(1) = \langle 2,\, 1,\, 2\rangle$, and, at time $t = 0$ it has velocity $\vec{\mathbf{v}}(0) = \langle 0,\, 0,\, 1\rangle$, compute $|\vec{\mathbf{r}}(2)|$, the magnitude of the position vector at $t = 2$.

A. $2\sqrt{53}$

B. $3\sqrt{21}$

C. $\sqrt{194}$

D. $\sqrt{293}$

E. $\sqrt{57}$

8. If $f(x, y) = x\sin(xy^2)$, then $f_{xy}(\pi, 1)$ is equal to

A. 4π

B. -4π

C. 2π

D. -2π

E. 0

9. Let $f(x,y,z)$ be a function which is differentiable at $(1,1,1)$ and

$$\frac{\partial f}{\partial x}(1,1,1) = -6, \qquad \frac{\partial f}{\partial y}(1,1,1) = 2, \quad \text{and} \quad \frac{\partial f}{\partial z}(1,1,1) = -1.$$

Let $\vec{\gamma}(t) = \langle x(t),\ y(t),\ z(t)\rangle$ be the parametric equation of a differentiable curve in $\mathbb{R}^3$ and suppose $\vec{\gamma}(0) = \langle 1,1,1\rangle$ and $\dfrac{d\vec{\gamma}}{dt}(0) = 3\vec{\mathbf{\imath}} - 3\vec{\mathbf{\jmath}} + \vec{\mathbf{k}}$. We can conclude that $\dfrac{d}{dt}f(\vec{\gamma}(t))$ at $t=0$ is equal to

A. -25

B. -14

C. -13

D. -11

E. -4

10. Which of the following is an equation for the plane tangent to the surface

$$z = \tan^{-1}(x^2+y^2) \quad \text{at the point} \quad \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{\pi}{4}\right)?$$

Hint: $\frac{d}{du}(\tan^{-1} u) = \frac{1}{1+u^2}$

A. $x + y - \dfrac{1}{2}z = \sqrt{2} - \dfrac{\pi}{8}$

B. $x + y - 2z = \sqrt{2} - \dfrac{\pi}{2}$

C. $x + y - \sqrt{2}z = \left(1 - \dfrac{\pi}{4}\right)\sqrt{2}$

D. $x + y - z = \sqrt{2} - \dfrac{\pi}{4}$

E. $x + y - 3z = \dfrac{\sqrt{2}}{3} - \dfrac{3\pi}{4}$

11. The absolute minimum value of $f(x,y) = 2 + x^2y^2$ in the region $\dfrac{x^2}{2} + y^2 \leq 1$ equals 2. The absolute maximum value of f in this region is

A. 4.5

B. 4

C. 3.5

D. 3

E. 2.5

12. Let $z(x,y)$ be the function implicitly defined as the solution to

$$x + y + z + \sin(xyz) = 3 + \frac{\pi}{2}$$

that satisfies $z(1,1) = \dfrac{\pi}{2}$. Find $z_x(1,1)$.

A. 1

B. -1

C. 2

D. 0

E. $\dfrac{3}{2}$