NAME ______________________________

STUDENT ID # ______________________________

INSTRUCTOR ______________________________

RECITATION INSTRUCTOR ______________________________

INSTRUCTIONS

1. There are 6 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. I.D.# is your 9 digit ID (probably your social security number). Also write your name at the top of pages 2–6.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. You need to show your work. Circle your answers in this test booklet for the first 10 questions.
4. No books, notes or calculators may be used on this exam.
5. Each problem is worth 10 points. The maximum possible score is 100 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 2, write 3802).
   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your student ID number, and fill in the little circles.
   (d) Using a #2 pencil, put your answers to questions 1–10 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
   (e) Sign your answer sheet.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your instructor.

1. The directional derivative of $f(x,y) = x^2 + 3xy - \frac{1}{2}\,y^2$ at $(-1,-1)$ in the direction of the vector $\vec{v} = 3\vec{i} + 4\vec{j}$ is

A. $\sqrt{5}$

B. $-\frac{23}{5}\,\sqrt{29}$

C. $-\frac{23}{7}$

D. 5

E. $-\frac{23}{5}$

2. Which of these vectors is perpendicular to the gradient of $f(x,y) = x^2y^3$ at $(1,2)$?

A. $4\vec{i} + 3\vec{j}$

B. $-2\vec{i} + \vec{j}$

C. $3\vec{i} - 4\vec{j}$

D. $-4\vec{i} + 3\vec{j}$

E. $\vec{i} - 2\vec{j}$

3. The point $(1,-1)$ is a critical point for $f(x,y) = x^3 + 3xy - y^3$.

   Which statement is true?

   A. $(1,-1)$ is a saddle point

   B. $f(1,-1)$ is a local minimum

   C. The 2nd derivative test gives no information about the nature of $f(1,-1)$.

   D. $f_{xx}(1,-1)f_{yy}(1,-1) < 0$

   E. $f(1,-1)$ is a local maximum

4. The tangent plane to the graph of the surface $z = e^{2x}\ln y$ at the point $(1,1,0)$ is

   A. $e^2y - z = e^2$

   B. $2e^2y - z = 1$

   C. $x - e^2y + z = 1$

   D. $2e^2x - e^2y = e^2$

   E. $2e^2x - e^2y + z = e^2$

5. The mass of the lamina that occupies the region D bounded above by $y^2 = x$ and below by $x^2 = y$ and has mass density $\rho(x, y) = y$ is

A. $\dfrac{1}{10}$

B. $\dfrac{3}{8}$

C. $\dfrac{3}{20}$

D. $\dfrac{1}{5}$

E. $\dfrac{4}{3}$

6. Find the <u>*area*</u> of the part of the plane $2x + 2y + z = 10$ that lies above the circle $x^2 + y^2 = 1$.

A. 3π

B. 2π

C. $\sqrt{8}\,\pi$

D. $\sqrt{10}\,\pi$

E. π

7. Let R be the triangle bounded by the lines $y = 2x,\ y = x + 4$ and the y–axis. Then

$$\iint_R f(x,y)dA = \int_0^a \int_b^c f(x,y)dydx \quad \text{where}$$

A. $a = 8,\ b = 0,\ c = \frac{y}{2}$

B. $a = 8,\ b = y - 4,\ c = \frac{y}{2}$

C. $a = 4,\ b = 2x,\ c = x + 4$

D. $a = 4,\ b = \frac{y}{2},\ c = 4$

E. $a = 4,\ b = x + 4,\ c = 2x$

8. The mass of the solid region bounded by the paraboloid $z = x^2 + y^2$ and by the plane $z = 1$ and which has mass density $\rho(x,y,z) = x^2 + y^2$ is

A. $\frac{8\pi}{15}$

B. $\frac{5\pi}{12}$

C. $\frac{16\pi}{15}$

D. $\frac{\pi}{6}$

E. $\frac{2\pi}{5}$

9. Compute the volume of the solid region bounded above by the sphere $\rho = 1$ and below by the cone $\phi = \dfrac{\pi}{3}$.

A. π

B. $\dfrac{2\pi}{9}$

C. 2π

D. $\dfrac{2\pi}{3}$

E. $\dfrac{\pi}{3}$

10. Let R be the image of the rectangle $S = [0,1] \times [0,2]$ under the transformation

$$x = 2u + 3v \quad , \quad y = u - v.$$

The area of R is

A. 15

B. 5

C. 10

D. 20

E. 14