MA 261 – Test 2

Name______________________________ **TA:** ______________________________________

Instructions

1. Each problem is worth 6 points except the last problem is worth 10 points.

2. Circle your choice of the correct answer and blacken the corresponding circle on the mark-sense sheet.

3. Calculators or books are not allowed.

4. Both the test booklet and the mark-sense sheet are to be given to the TA at the end of the examination.

1. The directional derivative of $f(x, y) = x^3 e^{-2y}$ in the direction of greatest increase of f at $x = 1$, $y = 0$ is

A. $3\vec{i}$

B. $3\vec{i} - 2\vec{j}$

C. 3

D. $\sqrt{5}$

E. $\sqrt{13}$

2. Find the minimum value of $f(x, y) = 2x + y$ subject to the constraint $x^2 + y^2 = 1$.

A. -1

B. $-\sqrt{5}$

C. -2

D. 0

E. $-\sqrt{3}$

3. Which of the following points corresponds to a local maximum of $f(x, y) = 6xy^2 - 2x^3 - 3y^4$.

A. $(0, 1)$

B. $(1, -2)$

C. $(1, 1)$

D. $(-1, 1)$

E. $(1, 0)$

4. Evaluate $\iint_R x^2 dA$, where R is the rectangle $0 \le x \le 3$, $0 \le y \le 2$.

A. 18

B. 9

C. 8

D. 19

E. $\frac{18}{3}$

5. Find the volume of the solid in the first octant bounded by $z = 4 - y^2$ and $x = 1$.

A. $\frac{13}{3}$

B. $\frac{20}{3}$

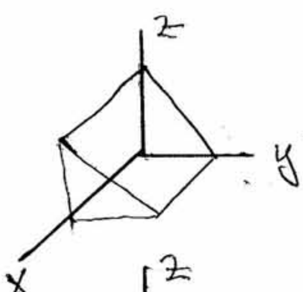
C. $\frac{18}{3}$

D. $\frac{16}{3}$

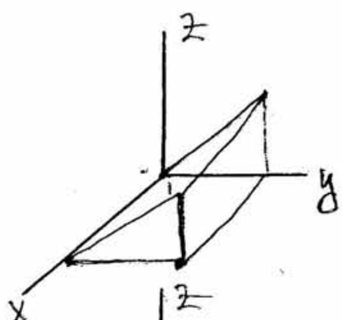
E. $\frac{22}{3}$

6. The integral $\int_0^1 \int_0^2 y \, dx dy$ is the volume of which region.

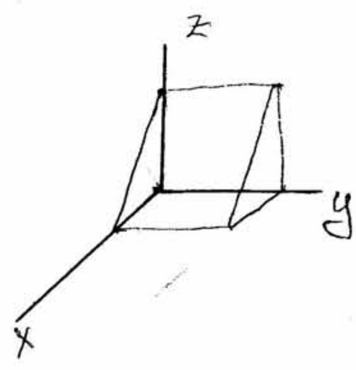
A.



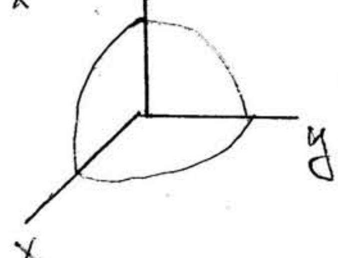
B.



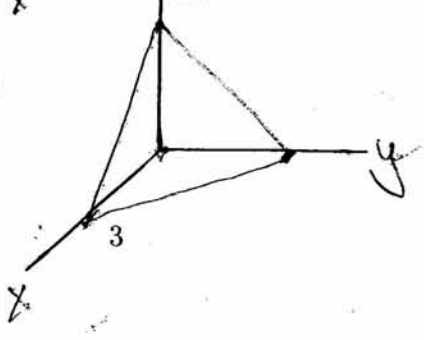
C.



D.



E.



7. The area of the triangle with vertices $(0, 2)$, $(0, 1)$, and $(1, 0)$ is given by

A. $\int_0^2 \int_{1-y}^{\frac{2-y}{2}} dxdy$

B. $\int_0^2 \int_x^{2x} dydx$

C. $\int_0^1 \int_x^{2x+1} dydx$

D. $\int_0^1 \int_{1-x}^{2-\frac{1}{2}x} dydx$

E. $\int_0^1 \int_{1-x}^{2-2x} dydx$

8. After an interchange of the order of integration the integral

$$\int_2^6 \int_1^{\frac{1}{2}x} f(x, y)dydx$$

equals

A. $\int_1^3 \int_{2y}^6 f(x, y)dxdy$

B. $\int_1^3 \int_2^{2y} f(x, y)dxdy$

C. $\int_1^3 \int_{\frac{x}{2}}^1 f(x, y)dxdy$

D. $\int_1^{\frac{x}{2}} \int_2^6 f(x, y)dxdy$

E. $\int_2^6 \int_1^{\frac{x}{2}} f(x, y)dxdy$

9. Evaluate $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} \cos(x^2+y^2)\,dy\,dx$.

A. $\frac{\pi}{2}\sin\frac{1}{2}$

B. $\pi\cos 1$

C. $\frac{\pi}{2}\sin 1$

D. $\pi\sin 1$

E. $\frac{\pi}{2}\cos 1$

10. Find the surface area of the part of the surface $z = x + y^2$ that lies above the triangle with vertices $(0,0)$, $(0,2)$, and $(2,2)$.

A. $\frac{13\sqrt{2}}{3}$

B. $\frac{26}{\sqrt{2}}$

C. $5\sqrt{5} - 21$

D. $\frac{16}{3}$

E. $\frac{17\sqrt{3}}{2}$

11. Evaluate $\int_{0}^{1}\int_{0}^{x}\int_{0}^{xy} (2x+8yz)\,dz\,dy\,dx$.

A. $\frac{10}{21}$

B. $\frac{3}{7}$

C. $\frac{11}{42}$

D. $\frac{9}{17}$

E. $\frac{12}{35}$

12. Find $\iiint_H z^3\sqrt{x^2+y^2+z^2}\,dV$ where H is the solid hemisphere with center the origin, radius 1, that lies above the xy-plane.

A. $\frac{3\pi}{7}$

B. 2π

C. $\frac{\pi}{14}$

D. $\frac{\pi}{25}$

E. $\frac{5\pi}{9}$

13. The region of integration of the iterated integral $\int_0^{\frac{\pi}{4}}\int_0^{3\sec\theta} r\,dr d\theta$ is

A. a rectangle

B. inside of part of a rose curve

C. inside of part of a cardioid

D. a triangle

E. a circular sector

14. Which integral gives the volume of the solid in the first octant bounded by the surfaces $x^2+z^2=9$, $y=2x$, $y=0$, $z=0$:

A. $\int_0^3\int_0^{\frac{y}{2}}\sqrt{9-y^2}dydx$

B. $\int_0^3\int_0^{2x}\sqrt{9-x^2}dydx$

C. $\int_0^6\int_0^{2x}(x^2+z^2)dydx$

D. $\int_0^6\int_0^{\frac{x}{2}}\sqrt{1-x^2}dydx$

E. $\int_0^3\int_0^{2x}(x^2+z^2)dydx$

15. Fill in the quantities a and b that convert the triple integral from rectangular coordinates to spherical coordinates:

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{x^2+y^2}} z\,dzdydx = \int_{0}^{\pi}\int_{a}^{\frac{\pi}{2}}\int_{0}^{2\csc\varphi} b\,d\rho d\varphi d\theta.$$

A. $a=0,\ b=\rho^3\sin\varphi\cos\varphi$

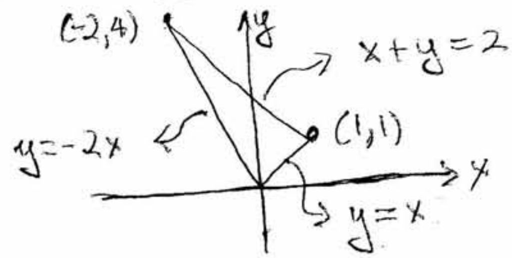
B. $a=0,\ b=\rho^2\cos\varphi$

C. $a=\frac{\pi}{4},\ b=\rho^3\sin\varphi$

D. $a=\frac{\pi}{4},\ b=\rho^3\cos\varphi\sin\varphi$

E. $a=0,\ b=\rho\cos\varphi$

16. Let R be the region in the xy-plane bounded by $y=-2x$, $y=x$, and $x+y=2$.



If $x=u-v$, $y=u+2v$, then $\iint_R (y-x)dA =$

A. $\int_0^1\int_0^{2-2u} 9v\,dvdu$

B. $\int_0^1\int_0^{2-2u} 3v\,dvdu$

C. $\int_0^1\int_0^{2} 9v\,dvdu$

D. $\int_0^1\int_{2-2u}^{2} 9v\,dvdu$

E. $\int_0^1\int_0^{\frac{(2-u)}{2}} 3v\,dvdu$