

Name \_\_\_\_\_

Student ID Number \_\_\_\_\_

Lecture \_\_\_\_\_

Recitation Instructor \_\_\_\_\_

Instructions:

1. This exam contains 10 problems, each worth 10 points.
2. Please supply all information requested above on the mark-sense sheet.
3. Work only in the space provided, or on the backside of the pages. Mark your answers clearly on the scantron. Also circle your choice for each problem in this booklet.
4. No books, notes, or calculator, please.

1. The set of all critical points of  $f(x, y) = x^2 + y^2 - x^2y + 4$  is
- A.  $\{(0, 0), (0, 1)\}$
  - B.  $\{(0, 0), (\sqrt{2}, 1)\}$
  - C.  $\{(\sqrt{2}, 1), (-\sqrt{2}, 1)\}$
  - D.  $\{(0, 0), (\sqrt{2}, 1), (-\sqrt{2}, 1)\}$
  - E.  $\{(0, 0), (0, 1), (\sqrt{2}, 1), (-\sqrt{2}, 1)\}$

2. The function  $f(x, y) = 2x^3 + xy^2 - 6x^2 + y^2$  has critical points at  $P(0, 0)$  and  $Q(2, 0)$ . Which of the following is true?
- A.  $f$  has a local max at  $P$  and a local min at  $Q$ .
  - B.  $f$  has a saddle point at  $P$  and a local max at  $Q$ .
  - C.  $f$  has a saddle point at  $P$  and a local min at  $Q$ .
  - D.  $f$  has a local max at  $P$  and a saddle point at  $Q$ .
  - E.  $f$  has a local min at  $P$  and a local max at  $Q$ .

3. Find the surface area of the part of the plane  $\sqrt{2}x + y + z = 6$  that lies inside the cylinder  $x^2 + y^2 = 2$ .

- A.  $2\pi$
- B.  $4\pi$
- C.  $2\sqrt{2}\pi$
- D.  $3\sqrt{2}\pi$
- E.  $3\pi$

4. If  $D$  is the region between the curves  $y = x^2$  and  $y = 2x - x^2$  the value of  $\iint_D x \, dA$  is:

- A. 0
- B.  $\frac{1}{3}$
- C.  $\frac{1}{4}$
- D.  $\frac{1}{12}$
- E.  $\frac{1}{6}$

5. By reversing the order of integration, we get  $\int_0^2 \int_{x^2}^4 x^3(\sin y) dy dx = \int_0^a \int_b^c x^3(\sin y) dx dy$   
with:

A.  $a = 2, b = \sqrt{y}, c = 2$

B.  $a = 2, b = 0, c = y^2$

C.  $a = 4, b = 0, c = y$

D.  $a = 4, b = 0, c = \sqrt{y}$

E.  $a = 4, b = \sqrt{y}, c = 2$

6. The volume of the solid under the graph of  $f(x, y) = x + 2y$  and above the region bounded by  $x = 0, y = 0,$  and  $y = 1 - x$  is:

A.  $\frac{1}{4}$

B. 1

C.  $\frac{1}{2}$

D.  $\frac{5}{2}$

E. 2

7. Evaluate the integral

$$\iint_R e^{x^2+y^2} dA$$

where  $R$  is the region in the first quadrant bounded by  $y = 0$ ,  $y = x$ ,  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 9$ .

- A.  $\frac{\pi}{4}(e^4 - 1)$
- B.  $\pi(e^6 - e)$
- C.  $\pi(e^9 - e)$
- D.  $\frac{\pi}{2}(e^3 - 1)$
- E.  $\frac{\pi}{8}(e^9 - e)$

8. Find the coordinate  $\bar{x}$  of the center of mass of a lamina in the first quadrant bounded by  $x = 0$ ,  $y = 0$ , and  $x^2 + y^2 = 4$  whose density at  $(x, y)$  is equal to the distance to the origin.

- A.  $\frac{\pi}{2}$
- B.  $\frac{3}{\pi}$
- C.  $\frac{3}{2}$
- D.  $\frac{4}{\pi}$
- E.  $\frac{\pi}{\sqrt{3}}$

9. If we use the method of Lagrange multipliers to find the maximum of  $f(x, y) = 2x^2 - y^2$  subject to the constraint  $x^2 + y^2 = 1$ , the Lagrange multipliers  $\lambda$  that we find are:

A. only  $\lambda = 2$

B. only  $\lambda = 0$

C. only  $\lambda = -1$

D.  $\lambda = 2$  and  $\lambda = -1$

E.  $\lambda = 0$  and  $\lambda = -1$

10. Find the volume of the solid region bounded below by  $z = \sqrt{x^2 + y^2}$  and on the top by  $x^2 + y^2 + z^2 = 1$ .

A.  $(2 - \sqrt{2})\frac{\pi}{3}$

B.  $\frac{2\pi}{3}$

C.  $\frac{4\pi}{3}$

D.  $(2 - \sqrt{3})\frac{\pi}{2}$

E.  $\frac{3\pi}{4}$