NAME ______________________________

STUDENT ID # ______________________________

INSTRUCTOR ______________________________

RECITATION INSTRUCTOR ______________________________

INSTRUCTIONS

1. There are 11 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. I.D.# is your 9 digit ID (probably your social security number). Also write your name at the top of pages 2–11.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. You need to show your work. Circle your answers in this test booklet for all 20 questions.
4. No books, notes or calculators may be used on this exam.
5. Each problem is worth 10 points. The maximum possible score is 200 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   - (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   - (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 2, write 3802).
   - (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your student ID number, and fill in the little circles.
   - (d) Using a #2 pencil, put your answers to questions 1–20 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect.
   - (e) Sign your answer sheet.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your instructor.

1. Find the point where the line

(L) $\vec{r}(t) = \langle 1 + 2t,\ -2 + 3t,\ 3 - t\rangle$

intersects the plane

(P) $5x - 2y - 3z = 0$

A. $(0, 0, 0)$

B. $(1, -2, 3)$

C. $(2, 3, -1)$

D. $(1, 1, 1)$

E. (L) does not intersect (P)

2. Find the cosine of the angle between $\vec{v} = \vec{i} - 2\vec{j} + 2\vec{k}$ and the tangent vector to the curve $\vec{r}(t) = e^t\ \vec{i} + e^t\ \vec{j} + t\ \vec{k}$ at the point $(1, 1, 0)$.

A. $\frac{1}{3\sqrt{3}}$

B. $\frac{5}{3\sqrt{3}}$

C. $\frac{1}{3\sqrt{2}}$

D. $\frac{2}{3\sqrt{3}}$

E. $\frac{4}{3\sqrt{2}}$

3. In spherical coordinates, the surface

$$\rho^2(1-\cos^2\phi)=16$$

is a

A. half cone

B. plane

C. sphere

D. cylinder

E. saddle surface

4. The length of the curve

$$\vec{r}\,(t)=(1-2t^2)\vec{i}+4t\vec{j}+(3+2t^2)\vec{k},\ 0\le t\le 2$$

is given by which integral

A. $\int_0^2 16\sqrt{t^2+2}\,dt$

B. $\int_0^2 \sqrt{8t^4+4t^2}\,dt$

C. $\int_0^2 \sqrt{8t+4}\,dt$

D. $\int_0^2 4t\,dt$

E. $\int_0^2 4\sqrt{2t^2+1}\,dt$

5. Find the position vector $\vec{r}(1)$ of a particle with acceleration vector $\vec{a}(t) = -10\vec{k}$, initial velocity $\vec{v}(0) = \vec{i} + \vec{j}$ and initial position $\vec{r}(0) = \vec{i} + 2\vec{j}$.

A. $\vec{i} + 2\vec{j}$

B. $2\vec{i} + 3\vec{j} - 5\vec{k}$

C. $\vec{i} + \vec{j}$

D. $2\vec{i} - \vec{j}$

E. $\vec{i} + \vec{j} - 5\vec{k}$

6. The maximum value of $3x^2 + 2y^2 - 4y$ given that $x^2 + y^2 = 16$ is

A. 40

B. 46

C. 60

D. 52

E. 64

7. The smallest value of $x^2 + y^2 + \dfrac{8}{x^2y^2}$ is

A. $\dfrac{58}{9}$

B. $\dfrac{17}{2}$

C. 6

D. 10

E. $\dfrac{129}{8}$

8. If the directional derivative of $f(x, y) = \dfrac{8y}{x}$ at $(2, 1)$ in the direction of $a\vec{i} + 3\vec{j}$ is equal to zero, then a equals

A. 6

B. 4

C. 5

D. -2

E. -3

9. Suppose $z = f(x, y)$ where $x = t^2uv$ and $y = u + tv^2$. Given that $\dfrac{\partial z}{\partial x}(4, 3) = 4$ and $\dfrac{\partial z}{\partial y}(4, 3) = -1$ compute $\dfrac{\partial z}{\partial t}$ when $(t, u, v) = (2, 1, 1)$.

A. 6

B. 7

C. 12

D. 15

E. 16

10. The area of the image of the square $[0, 1] \times [0, 1]$ under the map $T(u, v) = (u^2 + v,\ 2v)$ is

A. 4

B. 6

C. 2

D. 1

E. 3

11. Find a and b such that

$$\int_0^1 \int_{x^2}^1 f(x,y)\ dy\ dx = \int_0^1 \int_a^b f(x,y)\ dx\ dy$$

A. $a = 1,\ b = x^2$

B. $a = \sqrt{y},\ b = 1$

C. $a = 1,\ b = \sqrt{y}$

D. $a = \sqrt{y},\ b = 1$

E. $a = 0,\ b = \sqrt{y}$

12. Use polar coordinates to compute the double integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \sin\left(\frac{\pi}{2}\ (x^2 + y^2)\right)\ dy\ dx$$

A. $\frac{\pi}{2}$

B. $\frac{1}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

E. $\frac{\sqrt{\pi}}{2}$

13. The volume of the solid bounded from above by the sphere $x^2+y^2+z^2=4$ and below by the cone $z = \dfrac{1}{\sqrt{3}}\sqrt{x^2+y^2}$ is $\left(\tan\frac{\pi}{3}=\sqrt{3}\right)$

A. $\dfrac{\pi}{3}$

B. $\dfrac{5\pi}{2}$

C. $\dfrac{\pi}{2}$

D. $\dfrac{\pi}{4}$

E. $\dfrac{8\pi}{3}$

14. The volume of the solid bounded from below by $z = x^2+y^2$ and from above by $x^2+y^2+z^2=2$ is given, in cylindrical coordinates, by

A. $\displaystyle\int_0^{2\pi}\int_0^{1}\int_{r^2}^{\sqrt{2-r^2}} r\,dz\,dr\,d\theta$

B. $\displaystyle\int_0^{2\pi}\int_0^{\sqrt{2}}\int_{r^2}^{\sqrt{2-r^2}} r\,dz\,dr\,d\theta$

C. $\displaystyle\int_0^{2\pi}\int_0^{2}\int_{r^2}^{\sqrt{2-r^2}} r\,dz\,dr\,d\theta$

D. $\displaystyle\int_0^{2\pi}\int_0^{1}\int_{0}^{\sqrt{2-r^2}} r\,dz\,dr\,d\theta$

E. $\displaystyle\int_0^{2\pi}\int_0^{2}\int_{0}^{\sqrt{2-r^2}} r^2\,dz\,dr\,d\theta$

15. If C is the circle $x^2 + y^2 = 2$ oriented counterclockwise, then $\int_C -x^2y\,dx + y^2x\,dy$ is

A. 0

B. 2π

C. $\frac{4\sqrt{2}}{3}\,\pi$

D. 4π

E. 8π

16. If $\nabla f(x,y) = (2x + y^2 - 3x^2y)\vec{i} + (2xy - x^3 + 3y^2)\vec{j}$ and $f(0,0) = 1$, then $f(1,1)$ is

A. 5

B. 4

C. 3

D. 0

E. -1

17. If S is the portion of the paraboloid $z = 1 - x^2 - y^2$, with $z \geq 0$, oriented by the upward unit normal $\vec{n}$ and $\vec{F}(x, y, z) = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$, then $\displaystyle\iint_S \vec{F} \cdot \vec{n}\, dS$ is

A. 0

B. $\frac{\pi}{2}$

C. π

D. $\frac{3\pi}{2}$

E. 2π

18. If C is the curve given by $\vec{r}(t) = t\,\vec{i} + t^2\,\vec{j} + t^4\,\vec{k}$, $0 \leq t \leq 1$, then $\displaystyle\int_C (x^2 - y)dx + (y^2 - z)dy + (z^2 - x)dz$ is

A. $\frac{19}{12}$

B. $\frac{11}{9}$

C. 0

D. $\frac{-7}{15}$

E. $\frac{-5}{11}$

19. If S is the portion of the cone $z = 2(x^2 + y^2)^{1/2}$ with $0 \le z \le 2$, then $\displaystyle\iint_S z dS$ is

A. $\dfrac{2\sqrt{5}}{3}\ \pi$

B. $\dfrac{4\sqrt{5}}{3}\ \pi$

C. $\dfrac{8\sqrt{5}}{3}\ \pi$

D. $\dfrac{16\sqrt{5}}{3}\ \pi$

E. $\dfrac{32\sqrt{5}}{3}\ \pi$

20. Let S be the sphere $x^2 + y^2 + z^2 = 1$ with outward orientation. If $\vec{F}(x, y, z) = (x + yz)\vec{i} + (y + zx)\vec{j} + (z + xy)\vec{k}$ then $\displaystyle\iint_S \vec{F} \cdot \vec{n}\ dS$ is

A. 0

B. $\dfrac{2}{3}\ \pi$

C. 2π

D. $\dfrac{4\pi}{3}$

E. 4π