MA 261 **FINAL EXAM** **December 13, 2005**

Name: ____________________________

Student I.D. #: ____________________________

Lecturer: ____________________________

Recitation Instructor: ____________________________

Instructions:

1. This exam contains 22 problems worth 9 points each.
2. Please supply all information requested above and on the mark-sense sheet.
3. Work only in the space provided, or on the backside of the pages. Mark your answers clearly on the scantron. Also circle your choice for each problem in this booklet.
4. No books, notes, or calculator, please.

Key: BCAE DCBA DCED CADB
ACAA DC

MA 261 Final Exam Name:________________________________

1. Compute the angle θ between $\mathbf{a} = \mathbf{i} + \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. $\cos^{-1}\left(\frac{\sqrt{3}}{6}\right)$

E. $\cos^{-1}\left(\frac{\sqrt{3}}{4}\right)$

2. Find the equation of the plane that contains the line $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + t(\mathbf{i} - \mathbf{j} + \mathbf{k})$ and that is parallel to the vector $\mathbf{v} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

A. $2x - 3y + z = -1$

B. $3x - y + z = 4$

C. $5x + 4y - z = 10$

D. $4x - y - z = -1$

E. $x + y - 2z = -3$

3. If the acceleration of a moving particle is $\mathbf{a}(t) = (2t+1)\mathbf{i} + 2t^2\mathbf{j} - 4t\mathbf{k}$ and its initial velocity is $\mathbf{v}(0) = -\mathbf{i} + \mathbf{k}$, then its velocity when $t = 1$ is:

A. $\mathbf{i} + \frac{2}{3}\mathbf{j} - \mathbf{k}$

B. $\mathbf{i} + \frac{5}{3}\mathbf{j} + 3\mathbf{k}$

C. $-\mathbf{i} + \frac{1}{3}\mathbf{j} + \mathbf{k}$

D. $-\mathbf{i} + \frac{2}{3}\mathbf{j} + 2\mathbf{k}$

E. $\mathbf{i} + \frac{2}{3}\mathbf{j} - 3\mathbf{k}$

4. A solid region E is defined by $0 < x < y,\ 0 < z < \sqrt{x^2+y^2},\ x^2+y^2+z^2 < 4$. In spherical coordinates E is defined by:

A. $0 < \theta < \frac{\pi}{4},\ 0 < \phi < \frac{\pi}{4},\ 0 < \rho < 2$

B. $0 < \theta < \frac{\pi}{4},\ \frac{\pi}{4} < \phi < \frac{\pi}{2},\ 0 < \rho < 2$

C. $0 < \theta < \frac{\pi}{4},\ 0 < \phi < \frac{\pi}{4},\ 0 < \rho < 4$

D. $\frac{\pi}{4} < \theta < \frac{\pi}{2},\ 0 < \phi < \frac{\pi}{4},\ 0 < \rho < 4$

E. $\frac{\pi}{4} < \theta < \frac{\pi}{2},\ \frac{\pi}{4} < \phi < \frac{\pi}{2},\ 0 < \rho < 2$

5. The arclength of the curve:
$x = 1 - 2t^2,\ y = 4t,\ z = 3 + 2t^2,\ 0 \le t \le 2,$
is given by the integral:

A. $\int_0^2 16\sqrt{t^2+2}\,dt$

B. $\int_0^2 \sqrt{8t^4+4t^2}\,dt$

C. $\int_0^2 \sqrt{8t+4}\,dt$

D. $\int_0^2 4\sqrt{2t^2+1}\,dt$

E. $\int_0^2 4\sqrt{t^2+1}\,dt$

6. A vector parallel to the tangent vector to the curve
$x = 4\sqrt{t},\ y = t^2 - 2,\ z = \dfrac{4}{t}$ at the point $P(4, -1, 4)$ on the curve is:

A. $4\mathbf{i} - 9\mathbf{j} + 4\mathbf{k}$

B. $8\mathbf{i} + 6\mathbf{j} + \mathbf{k}$

C. $2\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$

D. $4\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$

E. $3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

7. The plane tangent to the graph of $f(x, y) = 3xy^2 - x^2 - 4y$ where $x = 2$ and $y = 1$ is:

A. $x + 2y - z = 4$

B. $x - 8y + z = -8$

C. $2x - 2y + z = 0$

D. $-x + 4y - z = 0$

E. $2x - 8y - z = -4$

8. Evaluate $\lim_{(x,y)\to(0,0)} \dfrac{(x^3 + xy^2)e^{2x}}{x^2 + y^2}$.

A. 0

B. 1

C. e

D. e^2

E. The limit does not exist

9. Find the slope of the curve $x^2 - xy - 2y^3 = 0$ at the point $(x, y) = (2, 1)$.

A. $-\frac{2}{3}$

B. $\frac{4}{3}$

C. $-\frac{3}{8}$

D. $\frac{3}{8}$

E. $-\frac{8}{3}$

10. Let $\mathbf{u}$ be a unit vector that points in the same direction as $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$. If $f(x, y) = x^2 + xy - y^2$ compute the directional derivative $D_{\mathbf{u}} f(2, 1)$.

A. $\frac{16}{5}$

B. $\frac{12}{5}$

C. 3

D. $\frac{13}{5}$

E. 4

11. Which vector is perpendicular to the tangent plane of the surface $x^2y - 2xz + 2y = -6$ at the point $(2, 1, 3)$?

A. $4\mathbf{i} - \mathbf{j} + 3\mathbf{k}$

B. $\mathbf{i} + \mathbf{j} - 2\mathbf{k}$

C. $2\mathbf{i} - \mathbf{j} + \mathbf{k}$

D. $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$

E. $-\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$

12. Find the absolute maximum of $f(x, y) = 2x - 2y - x^2 - y^2$.

A. 3

B. 4

C. 0

D. 2

E. 1

13. If E is the solid in the first octant that is bounded on the side by the surface $x^2 + y^2 = 4$, and on the top by the surface $x^2 + y^2 + z = 4$ represent the volume of E as an integral.

A. $\int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^{4-x^2-y^2} 1dzdydx$

B. $\int_0^4 \int_0^{\sqrt{4-x^2}} \int_0^{4-z} 1dzdydx$

C. $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} 1dzdydx$

D. $\int_0^2 \int_0^{4-x^2} \int_0^{x^2+y^2} 1dzdydx$

E. $\int_0^2 \int_0^{\sqrt{x^2+y^2}} \int_0^{4-x^2-y^2} 1dzdydx$

14. If R is the planar region defined by $1 \leq x^2 + y^2 \leq 9$ and $x \geq 0$, compute $\iint_R \sqrt{x^2 + y^2} dA$.

A. $\frac{26\pi}{3}$

B. $\frac{13\pi}{3}$

C. 9π

D. 4π

E. 13π

15. If S is the part of the surface $z = \sqrt{3}y + x^2$ that lies above the rectangle $0 \leq x \leq 1,\ 0 \leq y \leq 2$, compute $\iint_S x\,dS$.

A. $\frac{4}{3}(5\sqrt{5}-1)$

B. $\frac{2}{3}(2\sqrt{2}-1)$

C. $\frac{1}{3}(5\sqrt{5}-1)$

D. $\frac{4}{3}(2\sqrt{2}-1)$

E. $\frac{8}{3}(2\sqrt{2}-1)$

16. Compute $\iiint_E z\,dV$, where E is the intersection of the ball $x^2 + y^2 + z^2 < 1$ with the first octant.

A. $\frac{\pi}{8}$

B. $\frac{\pi}{16}$

C. $\frac{\pi}{12}$

D. $\frac{\pi}{6}$

E. $\frac{3\pi}{8}$

17. The vector field $\mathbf{F}(x,y) = (y^3 + 3x^2)\mathbf{i} + (3y^2x + 1)\mathbf{j}$ is conservative. Find f so that $\nabla f = \mathbf{F}$.

A. $f(x,y) = xy^3 + x^3 + y$

B. $f(x,y) = 6x + 6y^2 + xy$

C. $f(x,y) = \frac{1}{4}y^4 + x^3 + y^3x + x$

D. $f(x,y) = xy^4 + 3x^3y$

E. $f(x,y) = \frac{1}{4}y^4 + 3x^2y + \frac{3}{2}x^2y^2 + x$

18. Evaluate $\int_C ydx + \cos y dy$ where C is the polygonal path from $(0,0)$ to $(1,0)$ to $(1,2)$ to $(0,2)$ to $(0,0)$.

A. -1

B. 0

C. -2

D. $-\frac{3}{2}$

E. $\frac{2}{3}$

19. Evaluate $\int_C 4dx + 3dy$ where C is given by $x = t^2,\ y = t^3,\ 0 \le t \le 1$.

A. 7

B. 12

C. 8

D. 4

E. 10

20. If $\mathbf{F} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and S is the intersection of the solid cylinder $x^2 + y^2 \le 1$ with the plane $2x + y - z = 1$, compute $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$ (using an upward pointing $\mathbf{n}$).

A. $-\pi$

B. $-\frac{3\pi}{2}$

C. 2π

D. 3π

E. $\frac{\pi}{2}$

21. If S is the cone $z = \sqrt{x^2+y^2}$, $0 \le z \le 2$, that is upward oriented, and if $\mathbf{F} = -y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$, compute $\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$.

A. 4π

B. 16π

C. -4π

D. 8π

E. -16π

22. Let $\mathbf{F} = 4x\mathbf{i} - z\mathbf{j} + x\mathbf{k}$. Compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is the union of the hemisphere $x^2+y^2+z^2 = 1$, $z \ge 0$, and the base given by $x^2+y^2 \le 1$, $z = 0$. (Use the outward–pointing normal.)

A. $\frac{2\pi}{3}$

B. $\frac{16\pi}{3}$

C. $\frac{8\pi}{3}$

D. $\frac{4\pi}{3}$

E. $\frac{-4\pi}{3}$