1. Let C be the curve given by $\vec{r}(t) = \langle 4\sqrt{t}, t, 5 - t^2 \rangle$ for $t > 0$. At what point does the tangent line to C at $(4, 1, 4)$ intersect the xy plane?

A. $(0, 1, 0)$

B. $(4\sqrt{5}.\sqrt{5}, 0)$

C. $(2, 1, 0)$

D. $(8, 3, 0)$

E. $(0, -1, 0)$

2. The arclength of the curve $\vec{r}(t) = 2t\vec{i} + t^2\vec{j} + (\ln t)\vec{k}$ for $2 \leq t \leq 4$ is:

A. $\dfrac{17}{4}$

B. $4 + \ln 2$

C. $16 + \ln 2$

D. $\dfrac{15}{4}$

E. $12 + \ln 2$

3. A particle moves in space with acceleration $\vec{a}(t) = e^t\vec{k}$ and initial velocity and position given by $\vec{v}(0) = \vec{0}$, $\vec{r}(0) = \vec{j} + \vec{k}$. Where is the particle at time $t = 2$?

A. $(1, 1, e^2)$

B. $(0, 1, e^2)$

C. $(0, 1, e - 1)$

D. $(1, 1, e^2 - 2)$

E. $(0, 1, e^2 - 2)$

4. Suppose that z is defined implicitly as a function of x and y by the equation

$$e^{yz} + \sin(\pi yz) - xyz = 0.$$

What is the value of $\frac{\partial z}{\partial x}$ at $(e, 1, 1)$?

A. $-\frac{1}{e}$

B. $\frac{1}{e}$

C. $-\frac{1}{\pi}$

D. $\frac{1}{\pi}$

E. $\frac{1}{e - \pi}$

5. The surface area of a rectangular box is given by the function

$$S(x, y, z) = 2xy + 2yz + 2xz$$

where x, y, z are its sides. These are measured as $x = 10$ cm, $y = 20$ cm, $z = 30$ cm with possible errors in measurements as much as 0.1 cm. Use differentials to estimate the maximum error in the calculated surface area.

A. 12 cm^2

B. 24 cm^2

C. 36 cm^2

D. 48 cm^2

E. 60 cm^2

6. Given $\vec{a} = \langle 1, -1, 2 \rangle$ and $\vec{b} = \langle 2, 1, 0 \rangle$, find t such that the vector $\vec{c} = \langle 5, t-1, 2 \rangle$ is perpendicular to $\vec{a} \times \vec{b}$.

A. $t = 1$

B. $t = 2$

C. $t = -1$

D. $t = -2$

E. $t = 0$

7. The intersection of the hyperbolic paraboloid $x^2 - y^2 - z - 1 = 0$ with the yz-plane consists of

A. a hyperbola and a parabola

B. a hyperbola

C. an ellipse

D. two lines

E. a parabola

8. Let $f(x, y) = \sqrt{x^2 + y}$. The equation for the tangent plane to $z = f(x, y)$ at $(2, 1)$ is

A. $2\sqrt{5}z - 4x - y = 1$

B. $2\sqrt{5}z - 4x - y = 10$

C. $2z - 2x - y = 1$

D. $2z - 2x - y = 10$

E. $2\sqrt{5}z - 2x - y = 9$

9. The critical points of $f(x, y) = 3x^3 + 3y^3 + x^3y^3$ are:

A. $(0, 0), (1, -1)$

B. $(0, 0)$

C. $(1, 1)$

D. $(0, 0), (-3^{1/3}, -3^{1/3})$

E. $(-3^{1/3}, -3^{1/3}), (1, 1)$

10. The directional derivative of the function $f(x, y) = 4xy + e^{xy}$ at the point $(0, 1)$ and in the direction of $\vec{v} = \langle 3, -4 \rangle$ is:

A. 3

B. 15

C. $\langle 5, 0 \rangle$

D. $\langle 3, -4 \rangle$

E. -15

11. If

$$z = \frac{1}{u^2+v}, \quad u(s,t) = t + s^2, \quad v(s,t) = \ln(t)$$

then $\frac{\partial z}{\partial t}$ is:

A. $\frac{-(1+\frac{1}{t})}{(t+s^2)^2+\ln t}$

B. $\frac{-(2(t+s^2)+\frac{\ln t}{t})}{(t+s^2)^2+\ln t}$

C. $\frac{-(2(t+s^2)+\frac{1}{t})}{((t+s^2)^2+\ln t)^2}$

D. $\frac{-(u+1)}{(u^2+v)^2}$

E. $\frac{-(2u+1)}{u^2+v}$