MA 26100
FINAL Green
December 11, 2019

NAME ______________________ YOUR TA'S NAME ______________________

STUDENT ID # ____________________ RECITATION TIME ____________________

Be sure the paper you are looking at right now is GREEN! Write the following in the TEST/QUIZ NUMBER boxes (and blacken in the appropriate spaces below the boxes): **00**

You must use a #2 pencil on the mark–sense sheet (answer sheet). On the mark–sense sheet, fill in your TA's name and the COURSE number. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. Fill in your four-digit SECTION NUMBER. If you do not know your section number, ask your TA. Sign the mark–sense sheet.

There are **20** questions, each worth 10 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the mark–sense sheet and the exam booklet when you are finished.

If you finish the exam before 2:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 1:20. If you don't finish before 2:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ______________________________________

STUDENT SIGNATURE: ______________________________________

1. Which of the following pairs of equations describes a pair of orthogonal planes?

A. $3x + 2y + z = 4$ and $x + y - 5z = -1$

B. $x - y + 2z = 1$ and $-3x + 3y - 6z = 10$

C. $2x - y + 3z = 0$ and $4x + 4y + z = 0$

D. $x = y$ and $y = z$

E. None of the above.

2. On which of the following types of quadric surface does the following parametrized curve

$$\mathbf{r}(t) = \langle t \sin(t),\, 3t^2,\, -t \cos(t) \rangle$$

lie?

A. cone

B. sphere

C. ellipsoid, but not a sphere

D. paraboloid

E. None of the above.

3. Calculate the arc length of $\mathbf{r}(t) = \langle 3\sin(2t),\ 4,\ 3\cos(2t)\rangle$ for $0 \le t \le \pi/3$.

A. π

B. 2π

C. $5\pi/3$

D. 6π

E. $-\pi/3$

4. Find the maximum rate of change of $f(x, y) = \sqrt{7 - x^2 - y^2}$ at the point $(-2, 1)$.

A. $3/\sqrt{2}$

B. $\sqrt{8}$

C. $\sqrt{10}/2$

D. $1/4$

E. $5/\sqrt{2}$

5. Find the equation of the tangent plane to the surface $x^2 - y^2 + z^2 + 2 = 0$ at the point $(1, 2, 1)$.

A. $2x - 4y + 2z = -6$

B. $2x - 4y + 2z = 4$

C. $x - y + z = 0$

D. $-x - y + z = -1$

E. $x - 2y + z = -2$

6. Let $f(x, y) = e^{x+3y-3} \sin(\pi xy)$. Find $\dfrac{\partial f}{\partial x}(1, 1)$.

A. $-\pi$

B. $e\pi$

C. $-e\pi$

D. $-e\pi^2$

E. $-e$

7. The temperature at the point (x, y) is given by $T(x, y) = x^3 y$. Find the rate of change of the temperature with respect to time t at $t = 2$ along the path: $\mathbf{r}(t) = \langle t, t^2 \rangle$ of a moving particle.

A. 48

B. 60

C. 64

D. 70

E. 80

8. Consider the function

$$f(x, y) = \frac{1}{4}x^4 + xy + \frac{1}{4}y^4 \text{ on } \mathbb{R}^2$$

Then the function

A. has one saddle point and two local minima.

B. has 4 critical points.

C. has an absolute maximum and absolute minimum.

D. is always positive and hence has absolute minimum of 0.

E. has one local maximum and two local minima.

9. By changing the order of integration, compute

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-y^2}dydx$$

A. 0

B. $\pi/4$

C. $1/3$

D. $2/3$

E. 1

10. Find the volume of the region bounded below by the surface $z = 2 - \sqrt{4 - x^2 - y^2}$ and above by the surface $z = 6 - x^2 - y^2$. (Hint: use cylindrical coordinates)

A. π

B. $\frac{40}{3}\pi$

C. $\frac{16}{3} + \pi$

D. $\pi(\frac{53}{6} - \sqrt{3})$

E. $\frac{11}{6}\pi$.

11. Compute the line integral $\int_C (4x^3 + y^3)\, ds$, where C is the line segment from $(0, 0)$ to $(1, 2)$.

A. $3\sqrt{5}$

B. 0

C. $\sqrt{5}\pi$

D. $5\sqrt{5}/4$

E. -5

12. Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle yz,\ xz,\ xy\rangle$ and the curve C is parametrized by $\mathbf{r}(t) = \langle t^2,\ t,\ t^3 - 3t\rangle$, $1 \leq t \leq 2$.

A. 0

B. 10

C. 8π

D. -16

E. 18

13. Compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$ for $\mathbf{F} = \langle y^2, \, xy \rangle$, where C is the curve bounding the rectangle with corners (0, 0), (2, 0), (0, 1), and (2, 1) oriented counterclockwise.

A. 0

B. 1

C. -1

D. $-3/2$

E. $2e^2 + 2$

14. Compute $\oint_C y^2 \, dx + x \, dy$, where the curve C is the boundary of the half-disk

$$R = \{(x, \, y) : \, x^2 + y^2 \leq 9 \text{ and } x \geq 0\}$$

with clockwise orientation.

A. 0

B. $9\pi/2$

C. 9π

D. $-9\pi/2$

E. -3π

15. Given a two-dimensional vector field $\mathbf{F}(x, y) = \langle x^2 + \frac{y}{x^2+y^2},\ x - \frac{x}{x^2+y^2} \rangle$, compute the value of the scalar curl of $\mathbf{F}(x, y)$ at the point $(2, 1)$.

A. 3

B. 1

C. $7/\sqrt{5}$

D. $4/\sqrt{5}$

E. $5/\sqrt{5}$

16. Find the surface area of the parametric surface

$$\mathbf{r}(u, v) = \langle 2u + 3v,\ 3u + v,\ 2 \rangle, \quad 0 \le u \le 2,\ 0 \le v \le 1.$$

A. $3\sqrt{2}$

B. 14

C. 4

D. 12

E. $4\sqrt{2}$

17. Let S be the part of the plane $y + z = 10$ that lies inside the cylinder $x^2 + y^2 = 1$. Compute $\displaystyle\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$ for $\mathbf{F}(x, y, z) = \langle x,\, 1 - y + e^z,\, y - e^z \rangle$ with S oriented by the upward normal.

A. $2e\pi$

B. $-\pi e^2$

C. -2π

D. π

E. $1 - 4\pi$

18. Consider $\mathbf{F} = \dfrac{\mathbf{r}}{|\mathbf{r}|^3}$, where $\mathbf{r} = \langle x,\, y,\, z \rangle$ and $|\mathbf{r}| = (x^2 + y^2 + z^2)^{1/2}$. Which one of the following is true

(i) $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path.

(ii) $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS = 0$ for any closed surface S that encloses the origin.

(iii) $div(\mathbf{F}) = 0$.

A. None of the above.

B. Only (i) and (ii).

C. Only (i) and (iii)

D. Only (ii) and (iii).

E. All of the above.

19. Let $\mathbf{F} = (y + z\cos(x))\,\mathbf{i} + (-x + z\sin(y))\,\mathbf{j} + (xye^z)\,\mathbf{k}$, compute

$$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS,$$

where S is the part of the graph of $z = f(x, y) = e^x\,(x^2 + y^2 - 36)$ below the xy-plane with downward pointing normal.

A. 72π

B. 36π

C. 0

D. -36π

E. -72π

20. Compute

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$$

the net outward flux of the vector field $\mathbf{F} = \langle x + y,\, y - z,\, xy + z\rangle$ across the surface S, which is the boundary of the solid bounded by $z = 0$, $y = 0$, $y + z = 2$, and $z = 1 - x^2$.

A. $-32/5$

B. $-32/15$

C. $-16/5$

D. $32/15$

E. $32/5$