MA261 Spring 2015 Final Exam, 8:00-10:00am

1. Find the arc length of the curve given by

$$\vec{r}(t) = 2t\vec{i} + (3\sin 2t)\vec{j} + (3\cos 2t)\vec{k}, \qquad 0 \le t \le \pi$$

A. 2π

B. $\sqrt{2}\pi$

C.$2\pi\sqrt{10}$

D.$2\pi\sqrt{3}$

E.$\sqrt{13}\pi$

2. For $t = 2$, find a set of parametric equations of the tangent line to

$$\vec{r}(t) = t^2\vec{i} + 3t^3\vec{j} + t^4\vec{k}$$

A. $x = 4 + 4t$
$y = 24 + 36t$
$z = 16 + 32t$

B. $x = 4 + 4t$
$y = 24 + 24t$
$z = 16 + 16t$

C. $x = 4 + 2t$
$y = 24 + 9t$
$z = 16 + 4t$

D. $x = 4t$
$y = 24t$
$z = 16t$

E. $x = 4 + 4t$
$y = 24 + 9t$
$z = 16 + 4t$

3. If $f(x, y) = \sin(x^2 + y^2)$, then f_{yx} equals

A.$-2x\sin(x^2 + y^2)$

B.$-4xy\sin(x^2 + y^2)$

C.$-4xy\cos(x^2 + y^2)$

D.$-4x^2y\sin(x^2 + y^2)$

E.$4x^2y\sin(x^2 + y^2)$

4. An equation of the tangent plane to the graph of $2y = z^3 + 3xz$ at $(1, 7, 2)$ is

A. $15(x-1) + 6(y-7) - (z-2) = 0$

B. $6(x-1) - (y-7) + 15(z-2) = 0$

C. $6(x-1) - 2(y-7) - (z-2) = 0$

D. $6(x-1) - 2(y-7) + 15(z-2) = 0$

E. $6(x-1) - 2(y-7) + 12(z-2) = 0$

5. The level curves of $f(x, y) = x - \dfrac{y^2 + 1}{x}$ are

A. parabolas

B. ellipses

C. circles

D. lines

E. hyperbolas

7. A particle moves with acceleration $\vec{a}(t) = e^{2t}\vec{k}$, initial velocity $\vec{v}(0) = \vec{i} + \vec{j} + \dfrac{1}{2}\vec{k}$, and initial position $\vec{r}(0) = \dfrac{1}{4}\vec{k}$. Where is the particle at time $t = 1$?.

A. $\left(0, 0, \dfrac{1}{4}e^2\right)$

B. $\left(0, 0, \dfrac{1}{2}e\right)$

C. $\left(1, 0, \dfrac{1}{4}e^2\right)$

D. $\left(1, 1, \dfrac{1}{4}e\right)$

E. $\left(1, 1, \dfrac{1}{4}e^2\right)$

7. Determine if the following 3 limits exist. If the limit exists give its value, if the limit does not exist write DNE.

I. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$

II. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4+y^2}$

III. $\displaystyle\lim_{(x,y)\to(2,1)} \frac{x^2}{x^2+y^2}$

A. I. DNE, II. DNE, III. DNE

B. I. DNE, II. $\frac{1}{2}$, III. DNE

C. I. DNE, II. DNE, III. $\frac{4}{5}$

D. I. DNE, II. $\frac{1}{2}$, III. $\frac{4}{5}$

E. I. $\frac{1}{2}$, II. DNE, III. $\frac{4}{5}$

8. If $y = y(x, z)$ is defined implicitly by the equation

$$xy + y^3 = 2zy - z^3 + 1$$

compute $\left.\dfrac{\partial y}{\partial z}\right|_{(x,y,z)=(1,0,1)}$

A. 3

B. 0

C. 1

D. 2

E. -2

9. Compute $\dfrac{\partial w}{\partial r}$ at $(r,s) = (1,0)$, given that $w = x^2 - \dfrac{1}{4}y^4$ with $x = r^3 + rs^3$ and $y = r^2 + se^{2s}$.

A. 2

B. 3

C. 4

D. 5

E. 6

10. The rate of change of $f(x,y) = e^{xy} + y^2 - x^2 + 3$ at $(2,0)$ in the direction from $(2,0)$ to $(8,8)$ is

A. -8

B. $-\dfrac{2}{5}$

C. -4

D. $-\dfrac{4}{5}$

E. 8

11. Given that $(0,0)$ and $(1,3)$ are critical points of the differentiable function f and given that $f_x(x,y) = y - 3x^2$ and $f_y(x,y) = x - \frac{1}{9}y^2$, then

A. $(0,0)$ is a saddle point; $(1,3)$ is a local maximum of f

B. $(0,0)$ is a local minimum of f; $(1,3)$ is a saddle point

C. $(0,0)$ is a local minimum of f; $(1,3)$ is a local maximum of f

D. $(0,0)$ is a saddle point; $(1,3)$ is a local minimum of f

E. $(0,0)$ is local maximum of f; $(1,3)$ is a saddle point

12. Evaluate $\iint_D y^2 dA$ where D is the triangle with vertices $(0,0),\ (1,1),\ (0,1)$.

A. $\frac{1}{3}$

B. $\frac{1}{6}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

E. $\frac{1}{4}$

13. Compute the area of the region of the plane $z + 2x + 2y = 12$ that lies in the first octant.

A. 54

B. 28

C. 108

D. 36

E. 64

14. Transform into cylindrical coordinates and evaluate

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^{1-x^2-y^2} \sqrt{x^2+y^2}\, dzdydx$$

A. $\dfrac{2\pi}{25}$

B. $\dfrac{\pi}{2}$

C. $\dfrac{2\pi}{15}$

D. $\dfrac{\pi}{5}$

E. $\dfrac{3\pi}{5}$

15. Compute $\iiint_T dV$ where T is the solid in the first octant bounded by the plane $x + 2y + z = 4$ and the coordinate planes.

A. $\frac{3}{4}$

B. $\frac{2}{3}$

C. $\frac{16}{3}$

D. $\frac{5}{4}$

E. $\frac{9}{4}$

16. True or false: the vector field $\vec{F} = \langle 2xzy + ye^{xy}, x^2z + xe^{xy}, x^2y \rangle$ is conservative.

A. TRUE

B. FALSE

17. Evaluate the line integral

$$\int_C z^2 dx \; + \; x^2 \, dy \; + \; y^2 \, dz$$

where C is the line segment from $(1, 0, 0)$ to $(4, 1, 2)$.

A. 19/3

B. 23/3

C. 29/3

D. 11

E. 35/3

18. Evaluate

$$\int_C y^3 \, dx - x^3 \, dy$$

where C is the positively oriented circle of radius 2 centered at the origin.

A. -12π

$B. -8\pi$

$C. 24\pi$

$D. -24\pi$

$E. 12\pi$

19. Evaluate

$$\iint_S y\, dS$$

where S is the portion of the cylinder $x^2 + y^2 = 3$ that lies between the planes $z = 0$ and $z = 3$.

A. 0

B. 1

C. 2

D. 3

E. 4

20. Use the divergence theorem to evaluate

$$\iint_S \vec{F} \cdot d\vec{S}$$

where $\vec{F} = \langle xy, -(1/2)y^2, z \rangle$ and the surface S consists of three pieces: $z = 4 - 3x^2 - 3y^2$, $1 \leq z \leq 4$ on the top, the cylinder $x^2 + y^2 = 1$, $0 \leq z \leq 1$ on the sides, and $z = 0$ on the bottom.

A. π

$B.2\pi$

$C.(3/2)\pi$

$D.(5/2)\pi$

$E.0$