INSTRUCTIONS:

1. **Do not open the exam booklet until you are instructed to do so.**
2. This exam has 20 problems in 11 different pages (including this cover page). Once you are allowed to open the exam, make sure you have a complete test.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet.
4. Each problem is worth 10 points for a maximum of 200 points. No partial credit.
5. Use a #2 pencil to fill in the required information in your scantron and fill in the circles.
6. Use a #2 pencil to fill in the answers on your scantron.
7. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY:

1. Students must obey the orders and requests by all proctors, TAs, and lecturers.
2. No student may leave in the first 20 min or in the last 10 min of the exam.
3. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
4. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
5. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

NAME ______________________________

STUDENT SIGNATURE ______________________________

STUDENT PUID # ______________ SECTION NUMBER ______________

RECITATION INSTRUCTOR ______________________________

1. Find an equation for the line through the point $(1,\ 2,\ -3)$, and parallel to to the line

$$x = 7 + 2\,t, \quad y = 2 - t, \quad z = 4 + 3\,t$$

A. $x = 1 + 7\,t,\ y = 2 + 2\,t,\ z = -3 + 4\,t$

B. $x = 2\,t,\ y = -t,\ z = 3\,t$

C. $x = 7 + t,\ y = 2 + 2\,t,\ z = 4 - 3\,t$

D. $x = 1 + 2\,t,\ y = 2 - t,\ z = -3 + 3\,t$

E. $x = 2 + t,\ y = -1 + 2\,t,\ z = 3 - 3\,t$

2. Find an equation of the plane that contains the point $(2,\ 1,\ 1)$ and the line

$$x = 1 + 3\,t, \quad y = 2 + t, \quad z = 3 + t.$$

A. $3\,x + y + z = 8$

B. $2\,x + y + z = 6$

C. $x - 7\,y + 4\,z = -1$

D. $x + 2\,y + 3\,z = 7$

E. $-x + y + 2\,z = 1$

3. What is the x-cordinate of the point where the curve defined by $\langle 2\,t,\, t-1,\, 1\rangle$ intersects the cone $z^2 = x^2 + y^2$ for $t > 0$?

A. $\frac{4}{5}$

B. $\frac{2}{5}$

C. $\frac{3}{10}$

D. $\frac{3}{5}$

E. $\frac{3}{4}$

4. If $f(x,\, y) = \frac{3x^2+yx}{x^2+y^2}$ for $(x,\, y) \neq (0,\, 0)$, let l be the limit of $f(x,\, y)$ as $(x,\, y) \to (0,\, 0)$ along the x-axis, and let m be the limit of $f(x,\, y)$ as $(x,\, y) \to (0,\, 0)$ along the line $y = x$. Then

A. $l = 3,\, m = 2$

B. $l = 0,\, m = 2$

C. $l = 0,\, m = \frac{3}{2}$

D. $l = 3,\, m = 3$

E. $l = \frac{1}{2},\, m = \frac{1}{2}$

5. The equation of the tangent plane to the surface $z = e^{(x^2+y^2-10)}$ at the point where $(x, y) = (1, 3)$ is:

   A. $z = x + y - 3$

   B. $z = x + y - 2$

   C. $z = 2ex + 6ey - 19e$

   D. $z = 2x + 6y - 19$

   E. $z = x + 3y - 7$

6. If $f(x, y) = x^8 y^2$, $x = 12u + 2v$, and $y = \sin(uv)$, what is $\frac{\partial f}{\partial v}$ when $u = \frac{1}{6}$ and $v = \pi$?

   A. $2(2\pi + 2)^7 + (2\pi + 2)^8$

   B. $12(2\pi + 2)^7 + \frac{1}{3}(2\pi + 2)^8$

   C. $12(2\pi + 2)^7 + \frac{3\pi}{2}\ (2\pi + 2)^8$

   D. $4(2\pi + 2)^7 + \frac{\sqrt{3}\pi}{2}\ (2\pi + 2)$

   E. $4(2\pi + 2)^7 + \frac{\sqrt{3}}{12}\ (2\pi + 2)^8$

7. If $x^2y^3 + yx^3 = 10$, use implicit differentiation to compute $\frac{dy}{dx}$ at $(1, 2)$.

A. $\dfrac{22}{13}$

B. $-\dfrac{22}{13}$

C. $\dfrac{23}{12}$

D. $-\dfrac{23}{12}$

E. $-\dfrac{1}{24}$

8. How many critical points does the function $f(x, y) = 6xy^2 - 2x^3 - 3y^4$ have?

A. none

B. one

C. two

D. three

E. more than three

9. Let D be the domain in the first quadrant bounded by the curves $y = 0$, $x = y^2$, and $x+y = 2$. Compute $\iint_D y dA$.

A. $\frac{16}{3}$

B. $\frac{11}{12}$

C. $\frac{8}{3}$

D. $\frac{1}{3}$

E. $\frac{5}{12}$

10. The volume of the region below the graph of $z = 16 - x^2 - y^2$ and above the graph of $z = 3x^2 + 3y^2$ is:

A. 4π

B. 8π

C. 16π

D. 32π

E. 64π

11. Find the formulas for a and b that convert the triple integral from rectangular coordinates to spherical coordinates:

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{x^2+y^2}} x\,dz\,dy\,dx = \int_{0}^{\pi}\int_{a}^{\frac{\pi}{2}}\int_{0}^{2\csc\phi} b\,d\rho\,d\phi\,d\theta$$

A. $a = \frac{\pi}{4}, b = \rho^3 \sin^2\phi\cos\theta$

B. $a = 0, b = \rho^3 \sin^2\phi\cos\theta$

C. $a = 0,\ b = \rho\sin\phi\cos\theta$

D. $a = \frac{\pi}{4}, b = \rho^3\sin\phi$

E. $a = 0, b = \rho^2\sin\phi\cos\theta$

12. Let E be the region in the first octant that is inside the sphere $x^2+y^2+z^2=1$ and bounded by the planes $y=0$, $z=0$, and $y=x$. Compute $\iiint_E z\,dV$.

A. $\dfrac{\pi}{64}$

B. $\dfrac{\pi}{32}$

C. $\dfrac{\pi}{16}$

D. $\dfrac{\pi}{8}$

E. $\dfrac{\pi}{4}$

13. If surface S is parametrized by $\vec{r}(u, v) = \langle u, v, u^2 v \rangle$, then a vector normal to S at $(1, 2, 2)$ is

A. $\langle 1, 2, 2 \rangle$

B. $\langle 1, 2, 4 \rangle$

C. $\langle 4, 1, 1 \rangle$

D. $\langle 1, 2, -2 \rangle$

E. $\langle 4, 1, -1 \rangle$

14. Let f be a scalar function and $\vec{F}$ a vector field in R^3. Which of the following expressions are meaningful?

(I.) $\text{grad}\, \vec{F}$

(II.) $(\text{grad}\, f) \times \vec{F}$

(III.) $\text{div}\, f$

(IV.) $\text{curl}\ \vec{F}$

(V.) $\text{curl}\left(\text{div}\ \vec{F}\right)$

A. II, IV, V

B. II, IV

C. I, IV

D. I, II, V

E. I, II, IV

15. Find a function $f(x, y, z)$ such that $\overrightarrow{\nabla f}(x, y, z) = \vec{F}(x, y, z) = \langle yz, xz, xy \rangle$ and use it to compute the line integral $\int_C \vec{F} \cdot d\vec{r}$ where $C = \{x = t^3,\ y = 1 + t^2,\ z = (1 + t)^2,\ 0 \le t \le 1\}$.

A. 3

B. 4

C. 5

D. 7

E. 8

16. Let $\vec{F}(x, y, z) = 2(y + z)\vec{i} + (z + x)\vec{j} + (x + y)\,\vec{k}$, then $\operatorname{curl} \vec{F}(x, y, z) =$

A. $\vec{0}$

B. $-\vec{j}$

C. $\vec{j} + \vec{k}$

D. $\vec{j} - \vec{k}$

E. $-\vec{j} + \vec{k}$

17. If S is the portion of the cone $z = \left(x^2 + y^2\right)^{1/2}$ with $0 \le z \le 1$, then $\int\int_S z\,dS$ is

A. $\frac{\sqrt{2}}{3}\,\pi$

B. $\frac{4\sqrt{2}}{3}\,\pi$

C. $\frac{2\sqrt{2}}{3}\,\pi$

D. $\frac{\sqrt{3}}{3}\,\pi$

E. $\frac{2\sqrt{3}}{3}\,\pi$

18. If $\vec{F}(x,\,y,\,z) = \vec{i} + 2\,\vec{j} + \vec{k}$ and S is the intersection of the solid cylinder $x^2 + y^2 \le 1$ with the plane $2\,x + y - z = 1$, compute $\int\int_S \vec{F} \cdot \vec{n}\,dS$ (using an upward pointing $\vec{n}$).

A. $-\pi$

B. $-3\,\pi$

C. $2\,\pi$

D. $3\,\pi$

E. $\frac{\pi}{2}$

19. Evaluate the outward flux $\iint\limits_S \vec{F} \cdot d\vec{S}$ where $\vec{F}(x, y, z) = 3x\vec{i} + 4x^2\vec{j} + y^2\vec{k}$ and S is the closed surface (sides, top, and bottom) of the cylinder $x^2 + y^2 = 1$ between the planes $z = 0$ and $z = 1$.

A 3π

B. 8π

C. 1

D. 2

E. 0

20. If S is that part of the paraboloid $z = 4 - x^2 - y^2$ with $z \geq 0$, $\vec{F}(x, y, z) = \langle -y, x, z \rangle$, and $\vec{n}$ is the upward unit normal on S, then $\iint_S (\text{curl } \vec{F}) \cdot d\vec{S}$ equals

A. 2π

B. 4π

C. 8π

D. 16π

E. 0