MA 26100
FINAL EXAM Form **01**
May 1, 2017

NAME ______________________ YOUR TA'S NAME ______________________

STUDENT ID # ____________________ RECITATION TIME ____________________

1. You must use a #2 pencil on the scantron

2. a. Write **01** in the TEST/QUIZ NUMBER boxes and darken the appropriate bubbles on your scantron.

b. **The color of your scantron MUST match the color of the cover page of your exam**

3. On the scantron sheet, fill in your TA's name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

5. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.

6. Sign the scantron sheet.

7. Fill in your name and your instructor's name on the question sheets above.

8. There are 20 questions, each worth 10 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20. Also circle your answers on the exam itself. Do all your work on the question sheets.

9. Turn in both the scantron sheets and the question sheets when you are finished.

10. If you finish the exam before 8:50 pm, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 8:50 pm, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

11. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams. **ANY talking or writing during this time will result in an AUTOMATIC ZERO.**
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

1. Find the equation of the plane that contains the point $(2, 1, 1)$ and that is perpendicular to the planes $2x + y - 2z = 2$ and $x + 3z = 4$.

A. $x - 3y - z = -2$

B. $x - 3y - z = 2$

C. $3x - 2y + z = 5$

D. $2x - y + z = 4$

E. $3x - 8y - z = -3$

2. Compute

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \sqrt{9 - x^2 - y^2}\, dy\, dx$$

A. $\dfrac{9\pi}{2}$

B. $\dfrac{7\pi}{2}$

C. $\dfrac{8\pi}{3}$

D. $\dfrac{4\pi}{3}$

E. 9π

3. Express the area of the surface given by the graph of

$z = y^3 \cos^2(x) - 1$ over the triangular region, D, with vertices $(1,0),(1,3),(4,3)$.

Do not evaluate.

A. $\displaystyle\int_1^4 \int_{y+1}^1 \sqrt{4y^6 \cos^2(x)\sin^2(x) + 9y^4 \cos^4(x) + 1}\, dxdy$

B. $\displaystyle\int_0^4 \int_1^{y+1} \sqrt{-2y^3 \cos(x)\sin(x) - 3y^2 \cos^4(x)}\, dxdy$

C. $\displaystyle\int_1^3 \int_{1-y}^0 \sqrt{4y^6 \cos^2(x)\sin^2(x) + 9y^4 \cos^4(x)}\, dxdy$

D. $\displaystyle\int_1^3 \int_0^{y-1} \sqrt{4y^6 \cos^2(x)\sin^2(x) + 9y^4 \cos^4(x)}\, dxdy$

E. $\displaystyle\int_0^3 \int_1^{y+1} \sqrt{4y^6 \cos^2(x)\sin^2(x) + 9y^4 \cos^4(x) + 1}\, dxdy$

4. Compute the volume of the solid inside $x^2 + y^2 + z^2 = 25$ and outside $x^2 + y^2 = 16$, and such that $y \geq 0$.

A. 9π

B. $6\sqrt{3}\pi$

C. 18π

D. $3\sqrt{3}\pi$

E. 7π

5. Let $\mathbf{F}$ be a vector field and f be a scalar field. Which of the following expressions are meaningful?

i. $\operatorname{curl} f$

ii. $\operatorname{div}(\nabla f)$

iii. $(\nabla f) \times \operatorname{div}\mathbf{F}$.

iv. $\operatorname{curl}(\operatorname{curl}\mathbf{F})$

A. i and ii only

B. i only

C. i, iii, and iv only

D. iii only

E. ii and iv only

6. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the line segment from $P(1,0)$ to $Q(0,2)$ and $\mathbf{F}(x,y) = \langle ye^{xy}, xe^{xy} + \cos y\rangle$ is a **conservative** vector field.

A. $1 + \sin 2$

B. $\sin 2$

C. $1 - \sin 2$

D. $-1 + \sin 2$

E. $-1 - \sin 2$

7. Use Green's Theorem to evaluate $\oint_C xy^2\,dx + 2x^2y\,dy$ where C is the triangle with vertices $(0,0)$, $(2,2)$, and $(2,4)$ with counterclockwise orinetation.

A. 8

B. 12

C. 16

D. 20

E. 6

For problems 8 and 9 let $f(x, y) = y^3 - 3y + 3x^2y$.

8. The function $f(x, y)$ has local extrema consisting of:

A. 2 saddle points, 1 local max, 1 local min

B. 2 local max, 2 local min

C. 1 saddle point, 2 local max, 1 local min

D. 1 saddle point, 1 local max, 2 local min

E. 1 local max, 3 local min

9. The absolute minimum value "m" and absolute maximum value "M" for $f(x, y)$ on

$$D = \{(x, y) : -2 \leq x \leq 2, 0 \leq y \leq 2\}$$

satisfy

A. $m = -2$, $M = 18$

B. $m = -2$, $M = 2$

C. $m = -18$, $M = 18$

D. $m = -2$, $M = 26$

E. $m = -26$, $M = 26$

10. The total surface area for a silo (base with radius r, cylindrical side with height h, hemispherical cap) is $A = 3\pi r^2 + 2\pi rh$. Suppose h and r are measured to be $h = 20$ feet and $r = 8$ feet to within an error of $\frac{1}{2}$ foot for h and $\frac{1}{4}$ foot for r. Use differentials to estimate the maximum possible error for A (in square feet).

A. 52π

B. 30π

C. 8π

D. 22π

E. 4π

11. The equation $xyz - 2x \ln y - z^2 = 2$ implicitly defines $z = z(x, y)$ near $(x, y, z) = (3, 1, 1)$. Find the tangent plane to $z(x, y)$ at $(3, 1)$.

A. $z + x - 3y = -2$

B. $z - x - 3y = 0$

C. $z - x + 3y = 1$

D. $2z - x + 4y = 2$

E. $z + x - 3y = 1$

12. Let S be a parameterized surface given by

$$\begin{aligned}\mathbf{r}(u,v) &= \langle x(u,v), y(u,v), z(u,v)\rangle,\\ x &= uv,\\ y &= u^2 - v^2,\\ z &= u + 2v.\end{aligned}$$

Which of the following is an upward pointing normal to S at $\mathbf{r}(1,2)$?

A. $\langle 8, -3, -10\rangle$

B. $\langle -8, 3, 10\rangle$

C. $\langle 4, 1, -8\rangle$

D. $\langle -4 - 1, 8\rangle$

E. $\langle 4, 1, 0\rangle$

13. A surface S is the graph of $z = h(x,y) = xy$ above a region D. If $\int\int_S xyz\, dS = \int\int_D g(x,y)\, dxdy$ then $g(x,y)$ is

A. $\sqrt{y^6 + x^6 + x^4y^4}$

B. $\sqrt{x^2y^4 + x^4y^2 + x^2y^2 + 1}$

C. $\sqrt{x^4y^6 + x^6y^4 + x^4y^4}$

D. $\sqrt{x^2 + x^4 + x^2y^2}$

E. $\sqrt{x^4y^6 + x^6y^4}$

14. If $z = z(x, y, t), x = x(u, v, w), y = y(u, v, w), t = t(u, v, w)$ and $z = xe^{ty}, x = u^2 + v, y = vw + u, t = w^2u$ find z_v at $u = -2$, $v = 1$, $w = 2$.

A. -79

B. 18

C. 42

D. 19

E. -41

15. Compute $\int\int\int_E z\,dV$ where E is the solid between $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 4$, and in the first octant.

A. 2π

B. $\frac{7\pi}{8}$

C. $\frac{3\pi}{4}$

D. $\frac{15\pi}{16}$

E. $\frac{5\pi}{16}$

16. Let C be the curve parameterized by $\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} + (2t - t^2)\mathbf{k}$. At what point does the tangent line to C at $-2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ intersect the yz-plane?

A. $\langle 0, \frac{1}{2}, \frac{3}{2} \rangle$

B. $\langle 0, 2, -2 \rangle$

C. $\langle 0, -1, 1 \rangle$

D. $\langle 0, 0, 4 \rangle$

E. $\langle 0, -1, -1 \rangle$

17. Compute the arclength of the curve C defined by

$$\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} + (\ln t)\mathbf{k} \text{ for } 1 \leq t \leq 2.$$

A. $3 + \sqrt{2}$

B. $3 + \sqrt{2}$

C. $2 + 2\ln 2$

D. $3 + \ln 2$

E. $2 + \frac{1}{2}\ln 2$

18. Compute $\int\int_S x\,dS$ where S is the triangular region with vertices $(1,0,0),(0,2,0)$ and $(0,0,4)$.

A. $\frac{2\sqrt{13}}{3}$

B. $\frac{2\sqrt{17}}{3}$

C. $\frac{2\sqrt{21}}{3}$

D. $\frac{\sqrt{21}}{3}$

E. $\frac{\sqrt{17}}{3}$

19. Let S be the semi-ellipsoid $z = 2\sqrt{1-x^2-y^2}$, oriented so that the normal $\mathbf{n}$ is upward pointing. Let

$$\mathbf{F}(x,y,z) = x^2\mathbf{i} + y^2\mathbf{j} + z^2 \tan xy\,\mathbf{k}.$$

Use Stokes' Theorem to to evaluate $\int\int_S \text{curl}\mathbf{F}\cdot\mathbf{n}\,dS$.

A. 2π

B. 4π

C. 0

D. $\frac{4\pi}{3}$

E. $\frac{2\pi}{3}$

20. Let $\mathbf{F}(x, y, z) = x^2yz\mathbf{i} + xy^2z\mathbf{j} + xyz^2\mathbf{k}$. Using the outward-pointing normal on the surface S of the solid box defined by $0 \le x \le 1$, $0 \le y \le 2$, $0 \le z \le 2$ evaluate $\int\int_S \mathbf{F} \cdot d\mathbf{S}$.

A. 12

B. 24

C. 9

D. 18

E. 6