MA 26100
FINAL EXAM
05/06/2025
TEST/QUIZ NUMBER: **2311**

NAME ______________________ YOUR TA'S NAME ______________________

STUDENT ID # ____________________ RECITATION TIME ____________________

You must use a #2 pencil on the scantron sheet. Write **2311** in the TEST/QUIZ NUMBER boxes and blacken in the appropriate digits below the boxes. On the scantron sheet, fill in your **TA**'s name for the INSTRUCTOR and **MA 261** for the COURSE number. Fill in whatever fits for your first and last NAME. The STUDENT IDENTIFICATION NUMBER has ten boxes, so use **00** in the first two boxes and your PUID in the remaining eight boxes. Fill in your three-digit SECTION NUMBER. If you do not know your section number, ask your TA. Complete the signature line.
There are **20** questions, each worth 5 points. Do all your work in this exam booklet and indicate your answers in the booklet in case the scantron is lost. Blacken in your choice for each problem, 1–20, on the scantron. Use the back of the test pages for scrap paper. Turn in both the scantron sheet and the exam booklet when you are finished.
You may not leave the room in the first 20 minutes. If less than ten minutes remain until the end of the exam period, STAY SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ____

STUDENT SIGNATURE: ____

1. Find the tangent plane to $(x-2)^2 + 4y^2 + z^2 = 5$ at the point $(2, 1, 1)$.

A. $2x + 4y + z = 9$

B. $y = z$

C. $y = 1$

D. $y = -\dfrac{z}{4}$

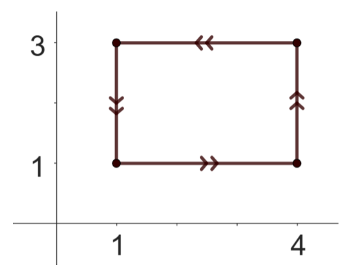
E. $y + z = x$

F. $4y + z = 5$

2. Let C be the (counterclockwise) boundary of the rectangle formed by the line segments from $(1, 1)$ to $(4, 1)$ to $(4, 3)$ to $(1, 3)$ to $(1, 1)$ (see the figure).

Compute the line integral

$$\oint_C \vec{\boldsymbol{F}} \cdot \, d\vec{\boldsymbol{r}}.$$



where $\vec{\boldsymbol{F}} = \langle xy, \; x^2 \rangle$.

A. 12

B. 3

C. 6

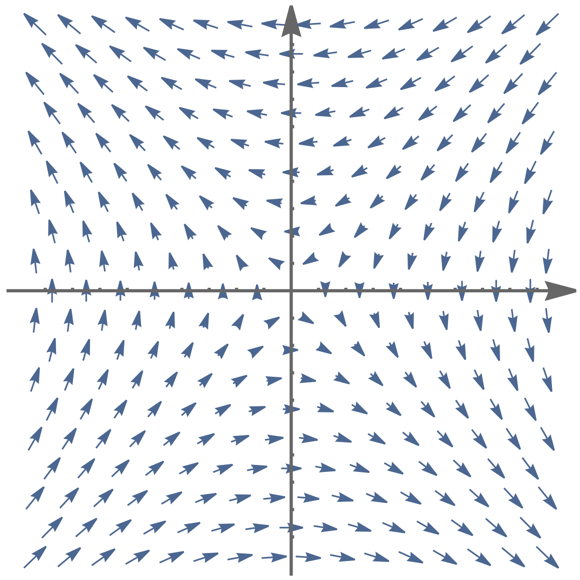
D. 0

E. 9

F. 15

3. This figure represents a vector field. Which one could it be?

A. $\vec{\boldsymbol{F}}(x,y) = -x\vec{\boldsymbol{\imath}} - y\vec{\boldsymbol{\jmath}}$
B. $\vec{\boldsymbol{F}}(x,y) = x\vec{\boldsymbol{\imath}} - y\vec{\boldsymbol{\jmath}}$
C. $\vec{\boldsymbol{F}}(x,y) = x\vec{\boldsymbol{\imath}} + y\vec{\boldsymbol{\jmath}}$
D. $\vec{\boldsymbol{F}}(x,y) = y\vec{\boldsymbol{\imath}} + x\vec{\boldsymbol{\jmath}}$
E. $\vec{\boldsymbol{F}}(x,y) = -y\vec{\boldsymbol{\imath}} - x\vec{\boldsymbol{\jmath}}$
F. $\vec{\boldsymbol{F}}(x,y) = y\vec{\boldsymbol{\imath}} - x\vec{\boldsymbol{\jmath}}$



4. Let $\vec{\boldsymbol{F}} = \left\langle \tan^{-1}\left(\frac{x}{y}\right),\ \ln(z-1),\ y^2 \right\rangle$. Compute $\nabla \cdot (\nabla \times \vec{\boldsymbol{F}})$ at $(0,1,2)$.

A. $\langle 0,\ 0,\ 0\rangle$
B. 0
C. 2
D. $\langle 1,\ 0,\ 0\rangle$
E. 1
F. $\langle 1,\ 0,\ 1\rangle$

5. The velocity of a moving object, for $t \geq 0$, is $\boldsymbol{r}'(t) = \langle 60,\, 96 - 32t \rangle$. Find $\boldsymbol{r}(1)$, if the initial position is $\boldsymbol{r}(0) = \langle 0,\, 3 \rangle$.

A. $\langle 0,\, -29 \rangle$
B. $\langle 60,\, 67 \rangle$
C. $\langle 0,\, -32 \rangle$
D. $\langle 60,\, 64 \rangle$
E. $\langle 60,\, 0 \rangle$
F. $\langle 60,\, 83 \rangle$

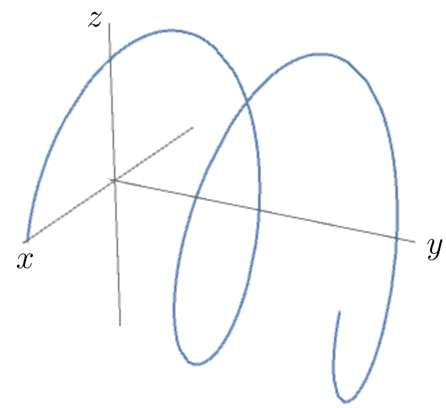
6. Let $\mathcal{S}$ be the part of the hemisphere $z = \sqrt{4 - x^2 - y^2}$ that is inside of the cylinder $x^2 + y^2 = 1$. If $\vec{\boldsymbol{F}} = \langle yz,\, -xz,\, xy \rangle$, compute the flux integral

$$\iint_{\mathcal{S}} \left(\nabla \times \vec{\boldsymbol{F}} \right) \cdot d\vec{\boldsymbol{S}}$$

where the normal vector to the surface $\mathcal{S}$ is oriented upward.
Hint: use Stokes' Theorem.

A. $-\dfrac{\sqrt{3}}{2}\pi$
B. $-2\pi\sqrt{3}$
C. $-\dfrac{\pi}{2}$
D. -2π
E. 0
F. $-\pi\sqrt{3}$

7. The graph below shows a curve on a circular cylinder. Which of the following vector valued functions could describe this curve?



A. $\vec{\boldsymbol{r}}(t) = \langle \cos(t), \sin(2t), \sin(t) \rangle$

B. $\vec{\boldsymbol{r}}(t) = \langle \cos(t), t, \sin(t) \rangle$

C. $\vec{\boldsymbol{r}}(t) = \langle \cos(t), \sin(t), t \rangle$

D. $\vec{\boldsymbol{r}}(t) = \langle t\cos(t), t, t\sin(t) \rangle$

E. $\vec{\boldsymbol{r}}(t) = \langle t, \sin(t), \cos(t) \rangle$

F. $\vec{\boldsymbol{r}}(t) = \langle \sin(t), \sin(2t), \cos(t) \rangle$

8. Compute the net outward flux of $\vec{\boldsymbol{F}} = \left\langle 2y^3(x^2+z^2),\ e^{x+z} - xy^4,\ 2z - \sqrt{x^2+y^2} \right\rangle$ across the sides of the box $\{(x,y,z) : 0 \le x \le 1,\ 0 \le y \le 2,\ 0 \le z \le 3\}$.
Hint: There is a better way to do this problem than by computing six flux surface integrals.

A. 22

B. 6

C. 12

D. 0

E. 36

F. 24

9. D is the ball of radius 1 centered at the origin. Compute the outward flux of $\vec{\boldsymbol{F}} = xy^2\vec{\boldsymbol{\imath}} + yz^2\vec{\boldsymbol{\jmath}} + zx^2\vec{\boldsymbol{k}}$ across the boundary of D.

A. $\dfrac{3\pi}{4}$

B. $\dfrac{4\pi}{5}$

C. π

D. $\dfrac{2\pi}{3}$

E. 0

F. $\dfrac{\pi}{2}$

10. Compute the line integral

$$\int_C -y\,dx + x\,dy + z\,dz$$

where C is the line segment from $(2, 1, 2)$ to $(8, 4, 4)$.

A. -24

B. 4

C. -10.5

D. 36

E. -2

F. 6

11. Consider the limit $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$. Which of the following statements is true?

A. The limit does not exist, because the path-restricted limit approaching $(0,0)$ along the diagonal $y = x$ does not exist.

B. The limit is 0, even though the path-restricted limits approaching $(0,0)$ along the x-axis and the y-axis are different.

C. The limit is 0, and the limit along any path approaching $(0,0)$ is also 0.

D. The limit does not exist, because the path-restricted limits approaching $(0,0)$ along the x-axis and the y-axis are different.

E. The limit is 0, because the path-restricted limit approaching $(0,0)$ along the diagonal $y = x$ is 0.

F. The limit does not exist, even though the path-restricted limits approaching $(0,0)$ along the x-axis and the y-axis are both 0.

12. Find the equation of the plane containing the points $(1,1,1)$, $(1,2,3)$, and $(3,1,0)$.

A. $x + 2y - z = 2$

B. $x - 4y + 2z = -1$

C. $2x + y + z = 7$

D. $2x - y + 4z = 5$

E. $2x - 2y + z = 1$

F. $x + 5y - z = 8$

13. Find the derivative of $f(x, y, z) = xyz$ in the direction $\vec{\imath} - 2\vec{k}$ at the point $(1, 2, 3)$.

A. $\dfrac{2}{\sqrt{5}}$

B. $\sqrt{5}$

C. 0

D. 5

E. 2

F. $\dfrac{\sqrt{5}}{2}$

14. Find the area of the surface $z = x^2 + y^2$ when $x^2 + y^2 \leq 1$.

A. $\dfrac{\sqrt{5}-1}{6}\pi$

B. $(\sqrt{5}-1)\pi$

C. $(5^{3/2}-1)\pi$

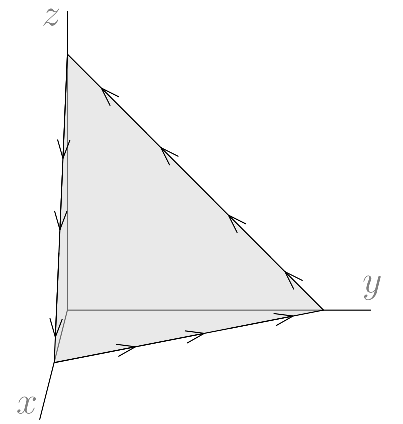
D. $\dfrac{\sqrt{2}+1}{6}\pi$

E. $(\sqrt{2}+1)\pi$

F. $\dfrac{5^{3/2}-1}{6}\pi$

15. Let C be the boundary of the triangle formed by the portion of the plane $z = 6 - 3x - y$ in the first octant, oriented as shown in the figure. Compute

$$\oint_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$$



for the vector field $\vec{\boldsymbol{F}} = \langle y + 2z, -x, -2x \rangle$.
Hint: use Stokes' Theorem.

A. 6

B. 3

C. 18

D. 9

E. 12

F. 0

16. Find the volume enclosed by the surfaces $z = x^2 + y^2$ and $z = 12 - x^2 - y^2$.

A. 18π

B. $2\sqrt{6}\pi$

C. $16\sqrt{6}\pi$

D. 36π

E. $12\sqrt{6}\pi$

F. 9π

17. Find the area of the region $D = \{(r, \theta) : \ 0 \leq \theta \leq 2\pi, \ 0 \leq r \leq 3 + \sin\theta\}$ in polar coordinates.
Hint: $\sin^2\theta = \dfrac{1 - \cos 2\theta}{2}$

A. 2π

B. 6π

C. $9\pi/2$

D. 9π

E. $19\pi/2$

F. 3π

18. Evaluate $\displaystyle\int_0^1 \int_{\sqrt{y}}^1 y e^{x^5} \, dx \, dy$

A. $\dfrac{e}{2}$

B. $\dfrac{e}{10}$

C. $\dfrac{e-1}{10}$

D. $\dfrac{e-1}{5}$

E. $\dfrac{e}{5}$

F. $\dfrac{e-1}{2}$

19. For vector field $\vec{\boldsymbol{F}} = \langle z,\ -y^2,\ 2x \rangle$, compute

$$\iint_{\mathcal{S}} \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{n}}\, dS,$$

the flux through the surface $\mathcal{S}$ given by the graph of $z = xy$ over the rectangle $0 \leq x \leq 3$, $-1 \leq y \leq 1$ with normal vectors oriented upward.

A. 18

B. 12

C. 3

D. 0

E. 9

F. 6

20. Find $\iint_{\mathcal{S}} f(x, y, z)\, dS$ where $f(x, y, z) = z + 4x$ and $\mathcal{S}$ is the surface given by $z = 8 - 4x - 8y$ with $x \geq 0,\ y \geq 0,\ z \geq 0$.

A. $16\sqrt{13}$

B. 48

C. $18\sqrt{3}$

D. 27

E. 60

F. $32\sqrt{5}$