MA 262, Fall 2017, Midterm 1
Version 01 (Green)

INSTRUCTIONS

(1) **Switch off your phone upon entering the exam room.**
(2) **Do not open the exam booklet until you are instructed to do so.**
(3) **Before you open the booklet, fill in the information below and us a # 2 pencil to fill in the required information on the scantron.**
(4) **MARK YOUR TEST NUMBER ON THE SCANTRON**
(5) Once you are allowed to open the exam, make sure you have a complete test. There are 6 different test pages with a total of 12 problems, plus this cover page.
(6) Do any necessary work for each problem on the space provided, or on the back of the pages of this booklet. Circle your answers in the booklet.
(7) Use a # 2 pencil to transcribe your answers to the scantron.
(8) After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY

(1) Do not leave the exam during the first 20 minutes of the exam.
(2) Do not leave in the last 10 minutes of the exam.
(3) No talking. Do not seek or obtain any kind of help from anyone to answer the problems on the exam. If you need assistance, consult an instructor.
(4) Do not look at the exam of another student. You may not compare answers with other students until your exam is finished and turned in, and then only after you have left the room.
(5) Your bags must be closed throughout the exam period.
(6) Notes, books, calculators and phones must be in your bags and cannot be used.
(7) Do not handle phones or cameras or any other electric device until you have finished and turned in your exam, and then only if you have left the room.
(8) When time is called, all students must put down their writing instruments immediately. You must remain in your seat while the TAs will collect the exam booklets and the scantrons.
(9) Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME _____

STUDENT SIGNATURE _____

STUDENT PUID _____

SECTION NUMBER _____

RECITATION INSTRUCTOR _____

(1) The general solution to the differential equation $y' = \frac{y}{x\ln(x)}$ is

(a) $y = \ln(x) + C$

(b) $y = x\ln(x) + C$

(c) $y = \ln(x)/x + C$

(d) $y = C \cdot \ln(x)$

(e) None of the above.

(2) The initial value problem

$$\frac{dy}{dx} + xy = e^{-x^2/2}, \qquad y(0) = 2$$

has as solution:

(a) $y = \frac{x+2}{e^{x^2/2}}$

(b) $y = x^2/2 + 2e^{x^2/2}$

(c) $y = 2(1+x) \cdot e^{x^2/2}$

(d) $y = 2e^{x^2/2}$

(e) $y = 2 - x \cdot e^{x^2/2}$.

(3) An integrating factor for the first order linear differential equation $y' + 2y/x = \ln(x)$ is
   (a) $I(x) = x^2$
   (b) $I(x) = x$
   (c) $I(x) = \ln(x)$
   (d) $I(x) = \ln(\ln(x))$
   (e) $I(x) = 1/\ln(x)$

(4) We consider the differential equation

$$(y^{-1} + x^{-1})dx + (xy^{-2} + 2y^{-1})dy = 0.$$

We hope that $I(x, y) = x^r y^s$ is an integrating factor for this equation. This will be the case if
   (a) $r = 1, s = 2$
   (b) $r = 0, s = 1$
   (c) $r = 2, s = 4$
   (d) $r = s = 1$
   (e) Actually, no matter how you choose r and s, the expression $x^r y^s$ will not be an integrating factor for this equation.

(5) The solution of

$$(4e^{2x} + 2xy - y^2)dx + (x - y)^2 dy = 0, \qquad y(0) = 3$$

satisfies the following equation:

(a) $2e^{2x} + x^2y - xy^2 = -(x - y)^3/3$
(b) $2e^{2x} + x^2y - xy^2 = 3$
(c) $4e^{2x} + 6x - 9 = (x - y)^2$
(d) $2e^{2x} + (x - y)^3/3 = 3$
(e) $2e^{2x} + x^2y - xy^2 + y^3/3 = 11$

(6) The solution $y(x)$ to the differential equation

$$y'' = -\frac{2}{1 - y}(y')^2, \qquad y(0) = 3, \qquad y'(0) = 4$$

is

(a) $y = 3 \cdot (1 - x)^3$
(b) $y = \frac{2x-3}{2x-1}$
(c) $y = \frac{4(1-x)^3+2}{3}$
(d) $y = -4\ln(1 - x) + 3$
(e) $y = 2\ln((1 - x)^2) + 2y^3/3$

(7) Suppose A is a 3×5 matrix, and $\vec{b}$ a column vector of length 3. You are told that row reduced echelon form of the augmented matrix $A^{\#}$ for the linear system $A\vec{x} = \vec{b}$ has three leading 1's, positioned in columns 1,4, and 6. Which of the following is true:

(a) it is not possible that the leading 1's are in the columns 1, 4 and 6
(b) A is invertible
(c) the system $A\vec{x} = \vec{b}$ is consistent
(d) the system $A\vec{x} = \vec{0}$ has infinitely many solutions
(e) none of the above

(8) Here a, b are real numbers. Let

$$A = \begin{bmatrix} a & 2 & b \\ a & a & 3 \\ a & a & a \end{bmatrix}$$

Given are the following statements:

(I) $\det(A)$ depends on the value of a but not on the value of b
(II) A is invertible as long as a is different from 2 and 3
(III) The row reduced echelon form of A has exactly two leading 1's for all real a, b

Of these statements, indicate which are correct:

(a) I and III
(b) II only
(c) II and III
(d) I only
(e) I and II

(9) The determinant of

$$A = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 2 & -3 & 0 & 2 \\ 0 & -1 & 1 & 1 \\ 2 & 1 & 0 & 2 \end{bmatrix}$$

is

(a) 0
(b) 24
(c) 32
(d) -8
(e) none of the above

(10) For two invertible matrices A and B of size $n \times n$, inspect the following statements:

(I) The sum of their two determinants is the determinant of their sum.
(II) The product of their two determinants is the determinant of their product.
(III) The sum of their two inverses is the inverse of their sum.
(IV) The inverse of AB is $A^{-1}B^{-1}$

Indicate below which are true for all choices of A and B:

(a) II only
(b) none at all
(c) all of them
(d) II and IV.
(e) I and III

(11) The inverse of $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{pmatrix}$ is

(a) $\begin{pmatrix} 1 & 0 & -1 \\ 1 & -1 & 2 \\ -1 & 1 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 1 & 2 \\ -1 & 1 & -1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & -1 \\ 1 & -1 & 2 \\ -1 & 1 & -1 \end{pmatrix}$

(d) $\begin{pmatrix} -1 & 0 & 1 \\ 1 & -1 & 2 \\ -1 & 1 & -1 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & 0 & -1 \\ 1 & -1 & 2 \\ 1 & 1 & -1 \end{pmatrix}$

(12) Let A be a 3×3 matrix with determinant 3. Then, the determinant of the adjoint matrix $\mathrm{adj}(A)$ of A is:

(a) $\det(\mathrm{adj}(A)) = 1$

(b) $\det(\mathrm{adj}(A)) = 3$

(c) $\det(\mathrm{adj}(A)) = 9$

(d) $\det(\mathrm{adj}(A)) = 27$

(e) In fact, the determinant of $\mathrm{adj}(A)$ cannot be determined from the information that is given.