MA262 — EXAM I — FALL 2023 — OCTOBER 2, 2023
TEST NUMBER 01– GREEN

INSTRUCTIONS:

1. **DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO AND MAKE SURE YOUR TEST AND YOUR SCANTRON ARE GREEN.**
2. Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.
3. **MAKE SURE YOU WRITE YOUR 10 DIGIT ID # AND YOUR TEST NUMBER ON YOUR SCANTRON.**
4. Once you are allowed to open the exam, make sure you have a complete test. **There are 8 different test pages including this cover page.**
5. There are eleven problems, each problem is worth 9 points and everyone gets one point. The maximum possible score is 100 points. No partial credit.
6. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. No extra paper is allowed. Circle your answers on this test booklet.
7. After you finish the exam, hand in your scantron and your test booklet to your professor, your TA or one of the proctors.

RULES REGARDING ACADEMIC DISHONESTY:

1. Do not leave the exam room during the first 20 minutes of the exam.
2. If you do not finish your exam in the first 50 minutes, wait until the end of the exam period to leave the room so you do not disturb your classmates.
3. Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructors.
4. Do not look at the exam of another student. You may not compare answers with anyone else or consult anyone other than your instructors until after you have finished your exam, handed it in to your instructor and left the room.
5. Do not consult notes, books, calculators.
6. Do not handle phones or cameras, or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the proctors will collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: _____ ID NUMBER: _____

SIGNATURE: _____

RECITATION SEC. NUMBER _____ TA's NAME: _____

1. If $y(x)$ satisfies the initial value problem

$$y' = \frac{x}{\sqrt{x^2+5}} \text{ and } y(2) = 2,$$

then $y(0)$ is equal to

A. $y(0) = -1$

B. $y(0) = 0$

C. $y(0) = \sqrt{2}$

D. $y(0) = \sqrt{5} + 1$

E. $y(0) = \sqrt{5} - 1$

2. If $y(x)$ satisfies the initial value problem

$$y'' + 4x = 0, \;\; y(0) = 1 \text{ and } y'(0) = 2,$$

then $y(1)$ is equal to

A. $y(1) = \dfrac{7}{3}$

B. $y(1) = \dfrac{3}{4}$

C. $y(1) = \dfrac{8}{3}$

D. $y(1) = \dfrac{7}{4}$

E. $y(1) = \dfrac{9}{4}$

3. The general solution to $y' = \dfrac{(x+2)y^2}{y^2 \cos y - 1}$ is implicitly given by

A. $\ln|\cos y - 1| = x^2 + 2x + C$

B. $\ln|y| + y = (x+2)^2 + C$

C. $\cos y = x^2 + 2x + C$

D. $\sin y - y = \dfrac{x^2}{2} + 2x + C$

E. $\sin y + \dfrac{1}{y} = \dfrac{x^2}{2} + 2x + C$

4. Let y be a solution of the initial value problem

$$y' - \frac{1}{2x}y = x, \quad y(1) = \frac{1}{3},$$

then $y(4)$ is equal to

A. 2

B. 4

C. 6

D. 8

E. 10

5. Initially a 100-gallon tank is **half full** of pure water. A salt solution containing 0.3lb of salt per gallon runs into the tank at a rate of 3 gallons per minute. The well mixed solution runs out of the tank at a rate of 4 gallons per minute. Let $A(t)$ be the amount of salt in the tank at time t. Then $A(t)$ satisfies the differential equation

A. $\dfrac{dA}{dt} = 0.9 - \dfrac{3A}{t+50}$

B. $\dfrac{dA}{dt} = 0.9 - \dfrac{4A}{50-t}$

C. $\dfrac{dA}{dt} = 0.9 + \dfrac{4A}{50-t}$

D. $\dfrac{dA}{dt} = 0.9 - \dfrac{4A}{t+50}$

E. $\dfrac{dA}{dt} = 0.9 - \dfrac{4A}{t+100}$

6. Which of the following is the implicit solution to the initial value problem

$$(e^x \sin y - y \sin x - 1)dx + (e^x \cos y + \cos x + 1)dy = 0,$$
$$y(0) = \pi?$$

A. $e^x \sin y + y \cos x + x - y = \pi$

B. $e^x \sin y + y \cos x + x - y = 0$

C. $e^x \sin y + y \cos x + y - x = \pi$

D. $e^x \sin y + y \cos x + y - x = 2\pi$

E. $e^x \sin y - y \cos x + y - x = 2\pi$

7. Let $y(x)$ solve the initial value problem

$$3y' + \frac{1}{x}y = 2y^{-2}, \quad y(1) = 1.$$

Then $y(27)$ is equal to

A. $y(27) = 5$

B. $y(27) = 7$

C. $y(27) = 3$

D. $y(27) = 4$

E. $y(27) = 6$

8. Find the solution to the differential equation

$$y' - \frac{y}{x} = \frac{x}{2y}, \quad y(1) = 3, \;\; x > 0.$$

A. $y(x) = x\sqrt{x\ln(x) + 9}$

B. $y(x) = x(\sqrt{\ln(x)} + 3)$

C. $y(x) = \sqrt{\ln(x) + 9x^2}$

D. $y(x) = x\sqrt{\ln(x) + 9}$

E. $y(x) = x\sqrt{\ln(x) + 9x}$

9. What can be said about the equilibrium solutions to the differential equation

$$\frac{dy}{dx} = y(y-1)(y-3)?$$

A. $y = 1$ is the only stable solution

B. $y = 0$ is the only stable solution

C. $y = 0$ and $y = 3$ are the only stable solutions

D. $y = 0$ and $y = 1$ are the only stable solutions

E. $y = 1$ and $y = 3$ are the only stable solutions

10. Consider the following system

$$\begin{aligned} x + 2y + z &= 1 \\ 7x + 4y + z &= 1 \\ 2x - y + mz &= a \end{aligned}$$

where m and a are real numbers. Only one of the following statements is correct. Which one is it?

A. If $m = 2$ and $a = 7$ the system has infinitely many solutions

B. If $m = -1$ and $a \neq -1$ the system has no solutions

C. If $m = 1$ and $a \neq 1$ the system has no solutions

D. If $m \neq -1$ and $a \neq -1$ the system has infinitely many solutions

E. The system has at least one solution for any a and m.

11. Which of the following is the reduced row echelon form of the matrix

$$\begin{bmatrix} 3 & 9 & -15 \\ 2 & 6 & -10 \\ 1 & 3 & -5 \end{bmatrix}?$$

A. $\begin{bmatrix} 1 & 3 & -5 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 3 & -5 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} \frac{1}{3} & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 3 & -5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 3 & -5 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$