

**MA262 — EXAM I — SPRING 2016 — FEBRUARY 25, 2016
TEST NUMBER 01**

INSTRUCTIONS:

1. **Do not open the exam booklet until you are instructed to do so.**
2. **Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
3. **MARK YOUR TEST NUMBER ON YOUR SCANTRON**
4. Once you are allowed to open the exam, make sure you have a complete test. There are 6 different test pages (including this cover page).
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet.
6. Each problem is worth 10 points. The maximum possible score is 100 points. No partial credit.
7. Use a # 2 pencil to fill in the answers on your scantron
8. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY:

1. Do not leave the exam room during the first 20 minutes of the exam.
2. If you do not finish your exam in the first 50 minutes, you must wait until the end of the exam period to leave the room.
3. Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructor.
4. Do not look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
5. Do not consult notes, books, calculators.
6. Do not handle phones or cameras, or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

STUDENT ID NUMBER: _____

SECTION NUMBER _____

RECITATION INSTRUCTOR: _____

1. The general solution of the differential equation $y' = x(1 + y^2)$ is:

A. $\ln(1 + y^2) = \frac{1}{2}x^2 + C$.

B. $y = \tan(\frac{1}{2}x^2 + C)$.

C. $y + \frac{1}{3}y^3 = \ln x + C$.

D. $\ln(1 + y^2) = \frac{1}{2}(x + C)^2$.

E. $y = \arctan(\frac{1}{2}x^2 + C)$.

2. An integrating factor of the differential equation $xy' + 3y = \ln x$ for $x > 0$ is:

A. $I(x) = x$.

B. $I(x) = x^2$.

C. $I(x) = x^3$.

D. $I(x) = \ln x$.

E. $I(x) = x \ln x$.

3. The solution of

$$y^{-2}dx - (2xy^{-3} + y)dy = 0, \quad y(1) = 1$$

satisfies the following equation:

A. $y^4 + y^2 - 2x = 0$

B. $y^4 - 2y^2 + x = 0$

C. $y^3 + y^2 - 2x = 0$

D. $y^4 + 2y^2 - 3x = 0$

E. $y^3 + y - 2x = 0$

4. Let $y(x)$ be the solution of the following initial value problem

$$y'' + 2y^{-1}(y')^2 = y', \quad y(0) = 1, \quad y'(0) = \frac{1}{3}.$$

Find $y(3)$.

A. $y(3) = e^3 + 1$

B. $y(3) = e^3$

C. $y(3) = 2e^2 + 1$

D. $y(3) = e + 1$

E. $y(3) = e$

5. The rank of $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 3 & 1 & 4 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{bmatrix}$ is equal to

A. 3

B. 4

C. 1

D. 2

E. 0

6. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ and let the inverse of A be denoted by $A^{-1} = [b_{jk}]$. Find b_{12} , which is the element in the first row and second column of A^{-1} .

A. $b_{12} = 2$

B. $b_{12} = 3$

C. $b_{12} = 4$

D. $b_{12} = 1$

E. $b_{12} = 5$

7. Let A be the 3×3 matrix of coefficients of the system $A\mathbf{x} = \mathbf{b}$. Given that the reduced row echelon form of the augmented matrix $(A|\mathbf{b})$ is equal to $\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. We can say that the solution set of the system $A\mathbf{x} = \mathbf{b}$ is given by

- A. $\{(2 - t, 2t, t), t \in \mathbb{R}\}$
- B. $\{(1 - t, 1 + 2t, t), t \in \mathbb{R}\}$
- C. $\{(1 + t, 1 + 2t, -t), t \in \mathbb{R}\}$
- D. $\{(1, 1, 0)\}$
- E. $\{(1 + t, 2 + 3t, t + 1)\}$

8. Given that $\det(A) = -8$ for $A = \begin{pmatrix} 2 & 3 & 0 & 1 \\ 1 & 3 & 2 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 \end{pmatrix}$ and $\alpha \in \mathbb{R}$, find the determinant of $\begin{pmatrix} 2 & 3 & 0 & 1 + \alpha \\ 1 & 3 & 2 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 \end{pmatrix}$.

- A. $8\alpha - 8$
- B. $4\alpha - 8$
- C. $-4\alpha - 8$
- D. $2\alpha - 8$
- E. $-2\alpha - 8$

9. Which of the following statements are correct?

I) The set of solutions to the differential equation $\frac{d^2y}{dx^2} + (1 + x^2)\frac{dy}{dx} + xy = 0$ is a subspace of $C^2(\mathbb{R})$.

II) The set $\{p(x) = x^2 + bx + c, b, c \in \mathbb{R}\}$ is a subspace of the space of all polynomials of degree two.

III) Let \mathbf{x} and \mathbf{y} be 3×1 column vectors. Let A be a 2×3 matrix, and suppose that $A\mathbf{x} = 0$ and $A\mathbf{y} = 0$. If $\mathbf{w} = 3\mathbf{x} - 2\mathbf{y}$, then $A\mathbf{w} = 0$.

A. I and III are true, but II is false

B. II and III are true, but I is false

C. I and II are true, but III is false

D. I, II and III are true

E. I, II and III are false

10. Let α and β be such that the vector $(3, \alpha, \beta)$ is in the span of $\vec{v}_1 = (5, 2, 1)$ and $\vec{v}_2 = (1, 0, 0)$. Then

A. $\alpha = -\beta$

B. $\alpha = 2\beta$

C. $\alpha = -3\beta$

D. $\alpha = 4\beta$

E. $\alpha = \beta$