

MA 262, Spring 2018, Midterm 1
Version 01 (Green)

INSTRUCTIONS

1. **Switch off your phone upon entering the exam room.**
2. **Do not open the exam booklet until you are instructed to do so.**
3. **Before you open the booklet, fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
4. **MARK YOUR TEST NUMBER ON THE SCANTRON**
5. Once you are allowed to open the exam, make sure you have a complete test. There are 6 different test pages with a total of 12 problems, plus this cover page.
6. Do any necessary work for each problem on the space provided, or on the back of the pages of this booklet. Circle your answers in the booklet.
7. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY

1. Do not leave the exam during the first 20 minutes of the exam.
2. No talking. Do not seek or obtain any kind of help from anyone to answer the problems on the exam. If you need assistance, consult an instructor.
3. Do not look at the exam of another student. You may not compare answers with other students until your exam is finished and turned in, and then only after you have left the room.
4. Your bags must be closed throughout the exam period.
5. Notes, books, calculators and phones must be in your bags and cannot be used.
6. Do not handle phones or cameras or any other electronic device until you have finished and turned in your exam, and then only if you have left the room.
7. When time is called, all students must put down their writing instruments immediately. You must remain in your seat while the TAs will collect the exam booklets and the scantrons.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME _____

STUDENT SIGNATURE _____

STUDENT PUID _____

SECTION NUMBER _____

RECITATION INSTRUCTOR _____

1. If y is the solution of the initial value problem

$$y' = x - y, \quad y(0) = 2,$$

then $y(2) =$

- A. $2 - 3e^{-2}$
- B. $1 + 2e^{-2}$
- C. $1 - e^{-2}$
- D. $3e^{-2}$
- E. $1 + 3e^{-2}$

2. If y is the solution of the initial value problem

$$\frac{dy}{dx} = \frac{2x(y-2)}{x^2+1}, \quad y(0) = 4,$$

then $y(1) =$

- A. 4
- B. 6
- C. 8
- D. 10
- E. 12

3. The general solution to the differential equation

$$y' = \frac{x^3 + 2y^3}{xy^2}, \quad x > 0$$

is

- A. $x^2 + y^3 = Cx$
- B. $y^2 = x^3 + x^6 + C$
- C. $y^2 = x^2 - x^6 + C$
- D. $x^3 + y^3 = Cx^6$
- E. $y^3 = Cx^3 + x^6$

4. Find the general solution to the differential equation

$$(y + \sin x)dx + (x + 2y - 3y^2)dy = 0.$$

- A. $2xy - \cos x + y^2 - y^3 = C$
- B. $xy - \sin x + 2y^2 - 3y^3 = C$
- C. $2xy - \cos x + 2y^2 - y^3 = C$
- D. $xy - \cos x + y^2 - y^3 = C$
- E. $2xy + \cos x + y^2 - y^3 = C$

5. A tank initially contains 10 gallons of water. A salt solution containing 2 pounds of salt per gallon runs into the tank at a rate of 3 gallons per minute. The well mixed solution runs out of the tank at a rate of 2 gallons per minute. Let $A(t)$ be the amount of salt in the tank at time t . Then $A(t)$ satisfies the differential equation

- A. $\frac{dA}{dt} = 6 + \frac{3A}{t+10}$
B. $\frac{dA}{dt} = 6 - \frac{2A}{5t+10}$
C. $\frac{dA}{dt} = 6 - \frac{2A}{t+10}$
D. $\frac{dA}{dt} = 4 + \frac{2A}{t+10}$
E. $\frac{dA}{dt} = 4 - \frac{2A}{5t+10}$

6. Consider the Bernoulli equation

$$\frac{dy}{dx} - \frac{2}{3x}y = 2y^3 \ln x.$$

After an appropriate substitution, the equation becomes

- A. $\frac{dv}{dx} + \frac{4}{3x}v = -4 \ln x$
B. $\frac{dv}{dx} + \frac{4}{3x}v = 2 \ln x$
C. $\frac{dv}{dx} + \frac{2}{3x}v = -2 \ln x$
D. $\frac{dv}{dx} - \frac{2}{3x}v = -2 \ln x$
E. $\frac{dv}{dx} - \frac{4}{3x}v = 2 \ln x$

7. If y is the solution of the initial value problem

$$y'' - \frac{1}{x}y' = 3x, \quad x > 0, \quad y(1) = 1, \quad y'(1) = 1,$$

then $y(2) =$

- A. 1
- B. 3
- C. 5
- D. 7
- E. 9

8. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}.$$

Then the $(3, 2)$ -entry of A^{-1} is:

- A. 0
- B. 1
- C. -1
- D. 2
- E. -2

9. Let $A = \begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix}$. Which one of the following statements is FALSE?

- A. For arbitrary 2×2 matrices B, C , if $BA = BC$ then $A = C$.
- B. A^T is invertible.
- C. For arbitrary 2×2 matrices B, C , if $AB = AC$ then $B = C$.
- D. $AA^{-1} = A^{-1}A$.
- E. A^{-1} is an upper triangular matrix.

10. For a real number a , consider the system of equations

$$\begin{array}{rclcl} x & + & y & + & z & = & 2 \\ 2x & + & 3y & + & 8z & = & 4 \\ 2x & + & 3y & + & (a^2 - 1)z & = & a + 1 \end{array}$$

Which of the following statements is true?

- A. If $a = 0$ the system is inconsistent.
- B. If $a = 2$ the system has infinitely many solutions.
- C. If $a = -2$ then the system has at least two distinct solutions.
- D. If $a = 3$ then the system has a unique solution.
- E. If $a = -3$ then the system is inconsistent.

11. Let A and B be $n \times n$ singular matrices. Which of the following statements must be true?

- (i) $\det(A) - \det(B) \neq 0$.
- (ii) $\det(A) + \det(B) = 0$.
- (iii) $\det(A + B) = 0$.
- (iv) $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions for every n -vector \mathbf{b} .
- (v) AB must be a singular matrix.

- A. (i) and (ii) only.
- B. (i) and (iii) only.
- C. (ii), and (v) only.
- D. (ii), (iv) and (v).
- E. (i), (ii), (iv) and (v).

12. Let

$$A = \begin{bmatrix} 4 & 0 & 1 \\ 1 & 2 & 3 \\ 0 & 2 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 & 1 & 0 \\ 3 & -5 & 2 \\ 0 & 0 & 2 \end{bmatrix}.$$

What is the determinant of

$$-2A \cdot B^{-1}?$$

- A. 7
- B. 14
- C. 21
- D. 28
- E. 56